Mathematical Modelling of Rhizosphere Processes

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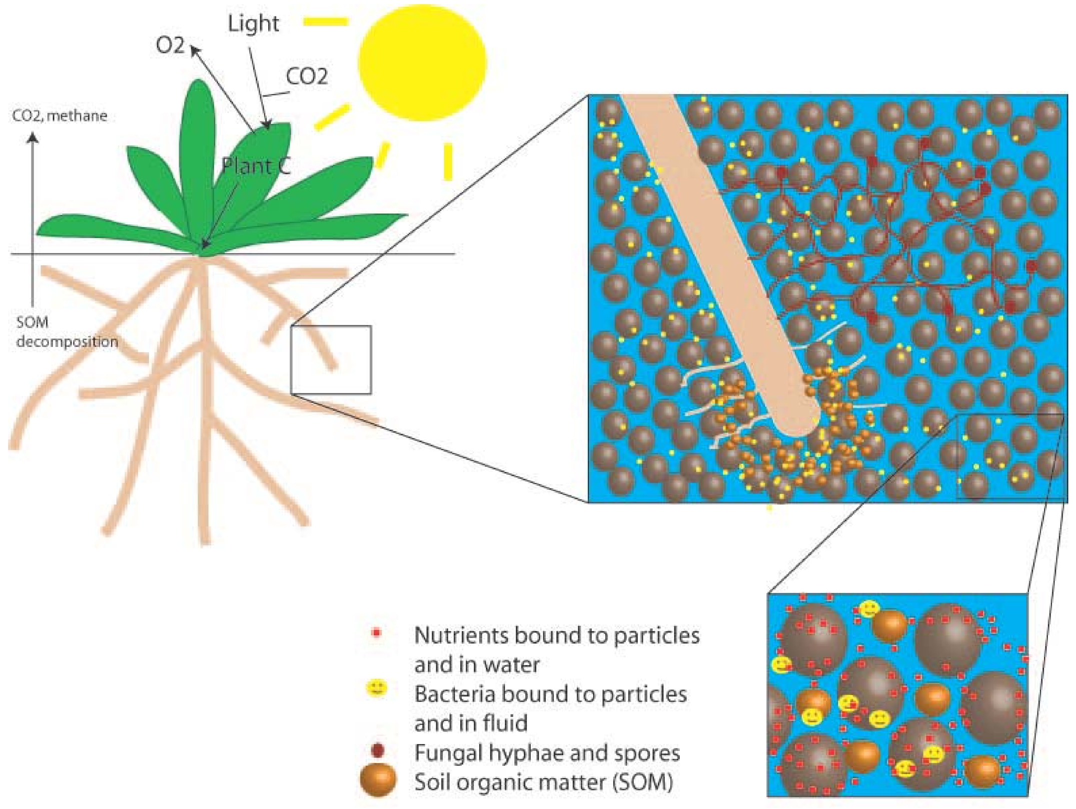

Figure 1. Spatial framework of plant–soil interaction. (a) The plant in the context of the soil–plant–atmosphere continuum, (b) some of the processes on a single-root scale and (c) the phenomena on the soil pore/particle scale (Roose and Schnepf, 2008).

Introduction

Our aim is a close collaboration between biological and mathematical communities for the development of quantitative mechanistic models describing rhizosphere processes. We believe that such models will lead to a more profound understanding of the fundamental science of plants and may help us with managing real-world problems such as food shortages and global warming. Figure 1 illustrates the variety of different spatial scales that must be considered. In the following we start at single root scale (1), we describe the root architecture (2); using up-scaling methods we can derive models for root system scale (3) and study the effect of different root responses (4).

1. Single Root Models

Water and solute movement around a single root is described in a mechanistic way using partial deferential equations. Solutes move in the soil according to the impeded convection diffusion equation. P solubilisation due to the presence of exudate is described by coupling P transport to the movement of exudate yielding coupled convection diffuson reaction equations. Figure 2 shows an illustration of relevant mechanisms that are included in the model. The impact of specific mechanisms can be determined using sensitivity analysis.

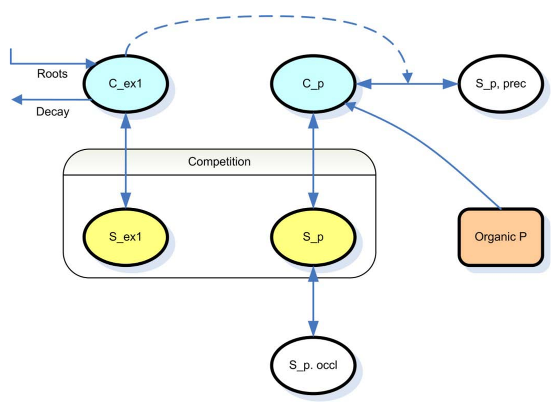

Figure 2. Illustration of mechanisms governing P availability

2. Dynamic Root Architecture Models

Root growth is fundamental in nutrient acquisition. We aim for an accurate dynamic description of the root system geometry. Figure 3 shows three examples using the root system model described in Leitner et al, 2010a. By using the geometric information together with the single root models we can up-scale and develop a root system model.

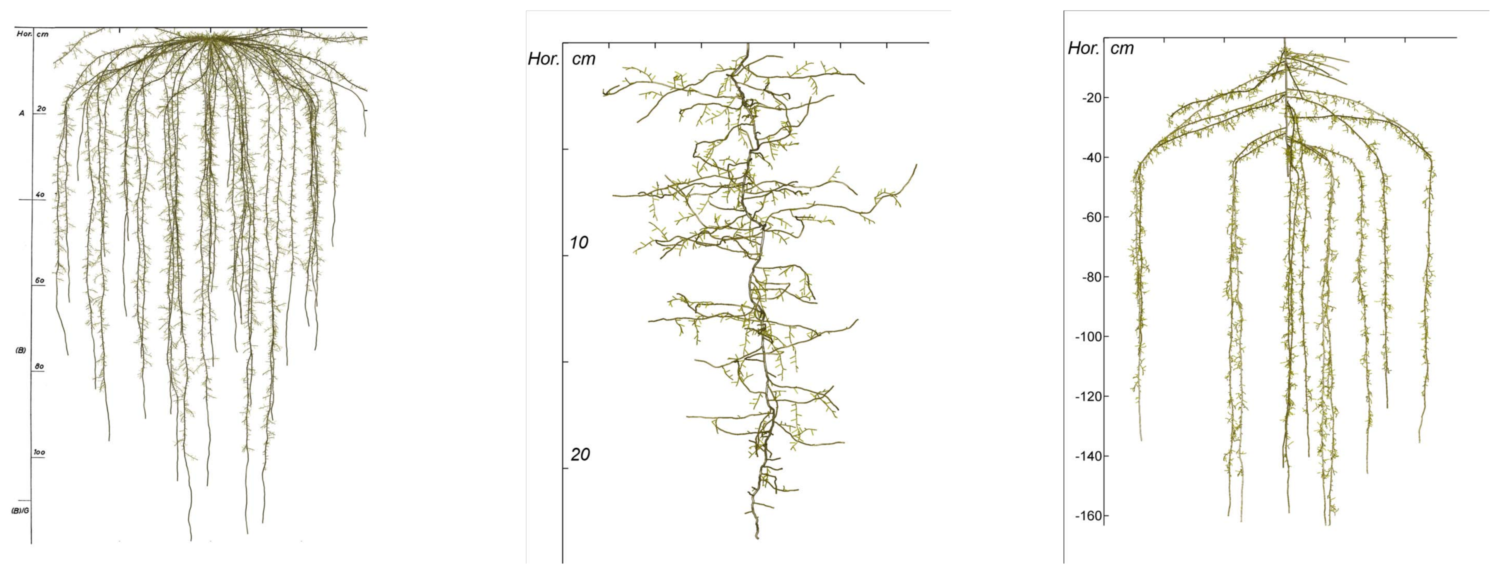

Figure 3. Simulated root systems (maize, anagallis, rape)

3. Up-scaling to root system scale

Single root models apply for individual root segments and the surrounding soil. The basic idea of up-scaling is to replace these microscopic equations that consider the complex root structure by effective equations that are obtained by homogenisation or averaging over a representative volume (Panfilov, 2000). New models are developed using formal multi-scale expansion on the microscopic equations. By taking the limit $\varepsilon \to 0$ we zoom out and obtain effective equations by reducing the geometric level of detail.

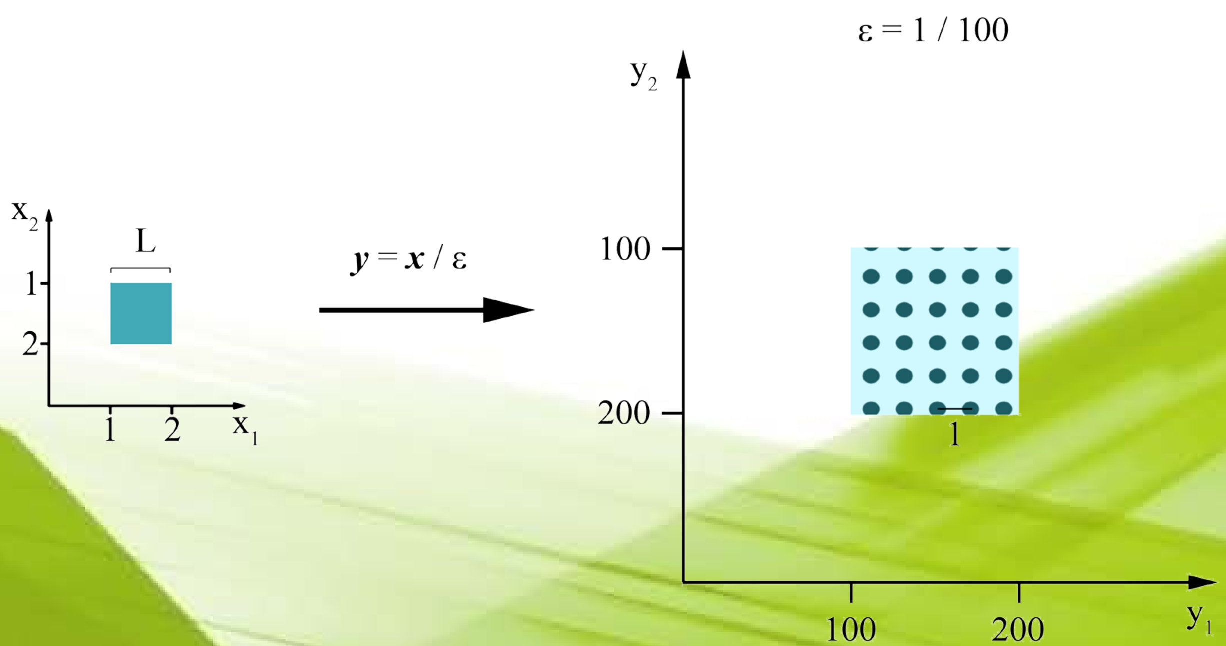

Figure 4.Illustrationof homogenisation across scales. The left-hand graph is in coarse ***x*** coordinates and shows the macrostructure of the system; the left-hand graph is in ***y*** coordinates and reveals the microstructure; The scaling parameter ε describes the reduction of the level of detail from micro- to macrostructure. (Leitner et. al, 2010b)

4. Root Responses

Root system architecture is controlled by local and systemic responses. For example, roots adapt to local perturbations like increased water content or P concentration. Globally, the root system adapts by systemic responses in dependence of internal plant P and water status. Figure 5 shows a simulation study of a root system with heterogeneous initial P concentration.

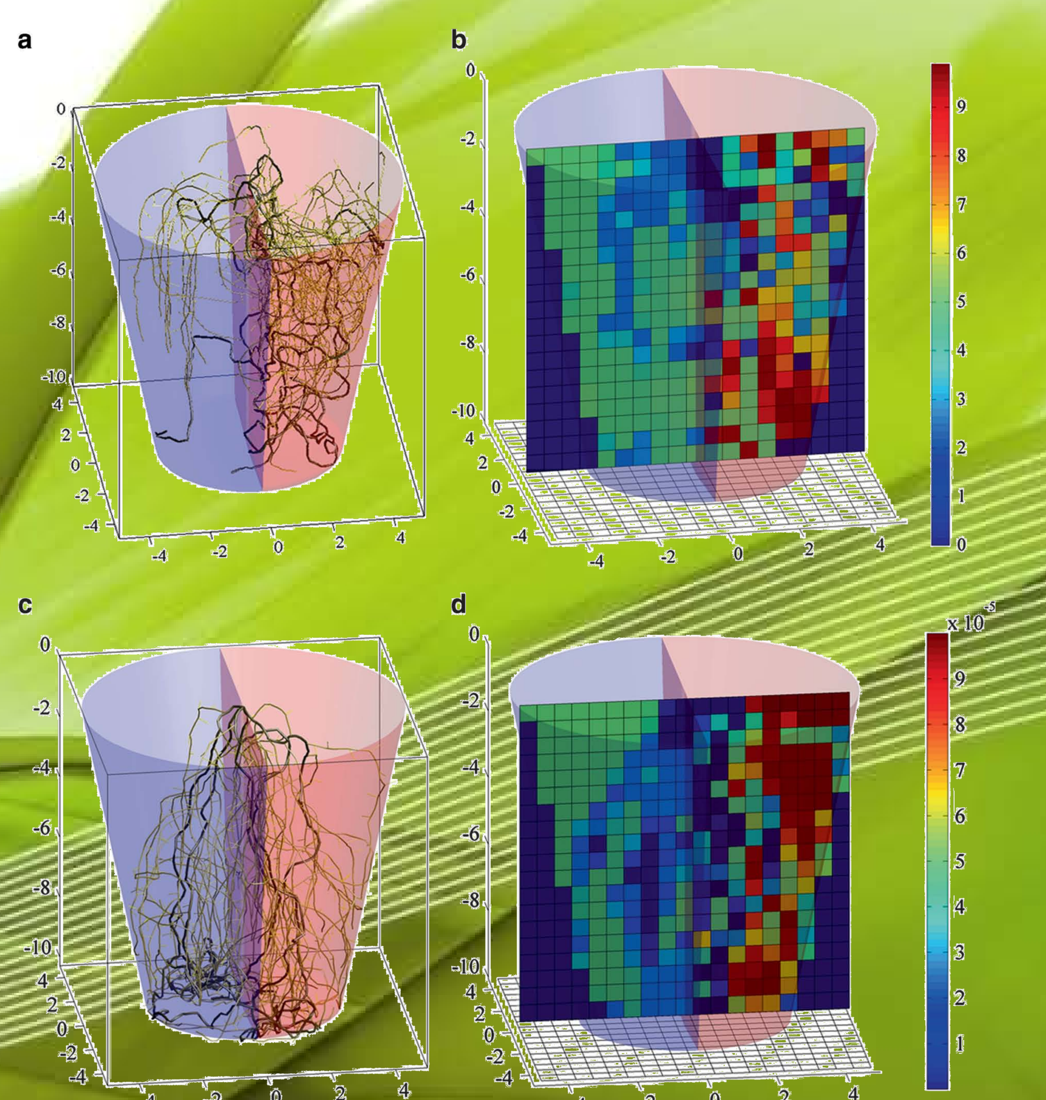

Figure 5. Root system development and soil P depletion after 20 days. Initial P concentration in pot 10^{-4} µmol cm^{-3} (right half) and $0.5*10^{-4}$ µmol cm^{-3} (left half). (a) Root system growth according to chemotropism. (b) Soil depletion due to root system shown in (a). (c) Root system growth according to gravitropism only. (d) Soil depletion due to root system shown in (c). (Schnepf et al, 2010)

References **Leitner**, D., Klepsch, S., Bodner, G., and Schnepf, A. (**2010a**). A dynamic root system growth model based on L-Systems. Plant and Soil, 332(1-2):177–192. **Leitner**, D., Klepsch, S., Ptashnyk, M., Marchant, A. Kirk, G., Schnepf, A., and Roose, T. (**2010b**). A dynamic model of nutrient uptake by root hairs. New Phytologist, 185(3):792–802. **Panfilov**, M. (**2000**). Macroscale Models of Flow Through Highly Heterogeneous Porous Media. Theory and Application of Transport in Porous Media. Kluwer Academic Publishers. **Roose**, T. **and Schnepf**, A. (**2008**). Mathematical models of plant-soil interaction. Phil. Trans. R. Soc. A, 366:4597–4611. **Schnepf**, A., D. Leitner, S. Klepsch, S. Pellerin, A. Mollier (**2010**). Modelling P dynamics in the soil-plant system. In: E. Bünemann, A. Oberson and E. Frossard (Eds.), Phosphorus in Action – Biological Processes in Soil Phosphorus Cycling. Springer-Verlag GmbH Berlin Heidelberg.