Incorporating Social Aspects in a Prospect Theory Model of Travel Choice

Erel Avineri
AFEKA, Tel-Aviv Academic College of Engineering
Avineri@Afeka.ac.il
Outline

• Gain/Loss asymmetry, reference dependency and Prospect Theory (PT)
• Application to network modelling
• Extending PT framework to represent
  – (i) heterogeneity;
  – (ii) dynamic and social reference dependency
• Numeric example to illustrate the model
• Emergence of learning? (simulation)
• Discussion and Conclusions
Loss Avoidance
Loss/Gain Asymmetry

• People are more sensitive to ‘bad outcomes’ than to ‘good outcomes’: The psychological effect of a loss is about twice than the psychological effect of a same-sized gain

• People tend to avoid losses more than they seek gains

• Gains and Losses are defined and measured against a “Reference Point” (Kahneman & Tversky, 1979; Thaler, 1985)
Prospect’s Theory Value Function

\[ v(x) = x^\alpha \]

\[ v(x) = -\lambda (-x)^\beta \]

\( \lambda \) Degree of loss aversion

\( \alpha, \beta \) degree of diminishing sensitivity

Reference Point

Gains

Losses
Travel time – Gain or loss?

travel as a derived demand
Travel time – Gain or loss?

Positive utility of the commute (Mokhtarian & Redmond, 2001)
Ideal positively related to actual and to ‘liking and utility’ of the commute
“…it is possible to commute as little as well as too much…”

“The gift of travel time” (Jain & Lyons, 2008)
YOUR TRAVEL TIME: 30 MIN
DESIRED TIME: 40 MIN
*** YOU WON! ***

http://atom.smasher.org/highway/
Reference Point: A travel choice context

Avineri & Bovy (2008)
Application of Prospect Theory to Network Modelling – Numeric (very simple!) example

Travel time is calculated for each route based on the formulation by the Bureau of Public Roads (BPR)

- 10 drivers choose their routes independently
- On route A, $t_f$ and $C$ are 10 (minutes) and 10 (vehicles per hour). On route B $t_f$ and $C$ are 12 and 5.
- $\alpha$ and $\beta$ were set to 0.5 and 1.0, for both routes
(‘Traditional’) User Equilibrium

Wardrop's principle of user equilibrium:

\[
Equilibrium \underbrace{\text{under condition that no user can}}_{\text{increase his/her route cost (time) by}} \underbrace{\text{unilaterally switching routes}}_{\text{by}}
\]

\[
t = t_f \left\{ 1 + \alpha \left( \frac{Q}{C} \right)^\beta \right\}
\]

\[
t_1 = 10 \left\{ 1 + 0.5 \left( \frac{Q_A}{10} \right) \right\} \quad t_2 = 12 \left\{ 1 + 0.5 \left( \frac{Q_B}{5} \right) \right\}
\]

\[
t_1 = t_2 = 14.1 \text{ min}
\]

\[
Q_A = 8.2
\]

\[
Q_B = 1.8
\]

\[
Q_A + Q_B = 10
\]

\[
t_1 = t_2 = 14.1 \text{ min}
\]
Application of Prospect Theory to Network Modelling

a user behaves as if s/he is a prospect maximizer; assumed to know the distribution of travel time on each route.

Wardrop's principle of user equilibrium can be extended:

"Equilibrium under condition that no user can increase his/her route prospect value by unilaterally switching routes"

\[ CWV_A^{Q_A} = CWV_B^{Q_B} \]

Equilibrium is reference-depended (Avineri 2006)
Application of Prospect Theory (and other reference-dependence models) to Network Modelling

- PT has been designed to capture individual ‘one-shot’ choices without feedback
- Homogeneity: All users share the same reference point
- Static: reference point(s) values are fixed over time

This work introduces:
- **Homogeneity** → **Heterogeneity** (random effects; individual experiences; social interactions – Festinger’s Theory of Social Comparisons)
- **Static** → **Dynamic** (imitation/learning processes)
Numeric Example

Behavioural assumptions of the model (1):

- **Gain/loss** (reference-dependency)
  \[ x_i = R P_i - T_i \]

- **Prospect-theoretic value** (following Tversky & Kahneman, 1979; \( \alpha = \beta = 0.88, \lambda = 2.25 \))

\( \lambda \) – degree of loss aversion;
\( \alpha, \beta \) – degree of diminishing sensitivity

\[ v = CWV(x) = \begin{cases} 
  x^\alpha & x \geq 0 \\
  -\lambda(-x)^\beta & x < 0 
\end{cases} \]
Numeric Example

Behavioural assumptions of the model (2):

- **Discrete choice rule:**

\[ p(switching) = 0 < (-0.25v + 0.5) < 1 \]

*examples:*  
\[ p(-15) = 0.875, \]
\[ p(+15) = 0.125, \]
\[ p(0) = 0.5 \]

- **Level of stability** of the network can be measured by the propensity of users to change route

\[ \sum_{i}^{N} p_{i}(switching) / N \]
Numeric Example

Behavioural assumptions of the model (3):

Updating the reference point:

- Initial values of RP (minutes) are random – uniformly distributed (heterogeneity)

\[ RP_i \in (10, 35) \]

\[ n=1 \text{ size of peer-group} \]

\[ RP_{i,t} = \text{Average}\{T_{i,t-1}, RP_{i,t-1}\} \]
Behavioral assumptions of the model (4):

Updating the reference point:

- Social learning/imitation of reference values

\[ n = 2 \text{ size of peer-group} \]

\[
RP_{i,t} = \text{Average} \left\{ T_{i,t-1}, \frac{\sum_{j=1}^{N} (RP_{j,t-1}x_{i,j})}{\sum_{j=1}^{N} x_{i,j}} \right\}
\]
Numeric Example

Behavioural assumptions of the model (4):

\[ \text{RP}_{i,t} = \text{Average}\left\{ T_{i,t-1}, \sum_{j=1}^{N} (\text{RP}_{j,t-1} x_{i,j}) \right\} \]

\[ \text{RP}_{1,t} = \text{Average}\{19, \frac{16+18}{2}\} = 18 \]

\[ \text{RP}_{2,t} = \text{Average}\{23, \frac{16+18}{2}\} = 20 \]

\[ \text{RP}_{1,t-1} = 16, T_{1,t-1} = 19 \]

\[ \text{RP}_{2,t-1} = 18, T_{1,t-1} = 23 \]
Numeric Example

Behavioural assumptions of the model (5):

Updating the reference point:

- Social learning/imitation of reference values

\( n=5 \) size of peer-group

\[
RP_{i,t} = \text{Average}\left\{ T_{i,t-1}, \sum_{j=1}^{N} (RP_{j,t-1}x_{i,j}) \right\} - \sum_{j=1}^{N} x_{i,j}
\]

\( x_j = \begin{cases} 
1 & i = j \text{ or } (i, j) \in \text{peer-group} \\
0 & \text{otherwise}
\end{cases} \)
Numeric Example

Behavioural assumptions of the model (6):

Updating the reference point:

- Social learning/imitation of reference values

   $n=10$ size of peer-group

\[
RP_{i,t} = \text{Average} \left\{ T_{i,t-1}, \sum_{j=1}^{N} (RP_{j,t-1}x_{i,j}) \right\} \frac{\sum_{j=1}^{N} x_{i,j}}{\sum_{j=1}^{N} x_{i,j}}
\]
Research Questions

• SOCIAL/DYNAMIC RP mechanism (as described in the numeric example) – Emergence of (social) **Learning**?

  - Convergence of **perceived RP** towards a specific value

  - Convergence of route choices and **travel times** towards a user equilibrium

  - **Stability** - propensity of users to change route is to be reduced over time (less fluctuations)

• Effect of the size of the peer-group ($n=1, 2, 5, 10$)?
Methodology

• Simulation \((20 \text{ runs})\) of perceived RPs and route choices of 10 agents over 12 rounds.

• Based on traffic assignment at the previous round, and previous values of travel times and reference points, values are updated for each agent.

• Repeated for different sizes of the peer-group \((n=1,2,5,10)\).

• Also – a model without loss aversion and diminishing sensitivity \((\alpha=\beta=0, \lambda=0)\), but with dynamic/social updating of reference points.

\((2*4 \text{ conditions}) \times (20 \text{ simulation runs}) \times (12 \text{ rounds}) \times (10 \text{ agents})\)
Results (1)
Reference point values
(20 runs, 10 agents, for each round, for each peer-group size)
Results (2)
Travel times
(20 runs, 10 agents, for each round, for each peer-group size)
Results (3)

Stability (propensity to change route)
(20 runs, 10 agents, for each round, for each peer-group size)
Results (4)

Reference point values
(20 runs, 10 agents, for each round, for each peer-group size)

Prospect Theory $\alpha = \beta = 0.88$, $\lambda = 2.25$

‘Traditional’ Utility model $\alpha = \beta = 0$, $\lambda = 0$

RP → 14.5

RP → 16.0
Results (5)
Travel times
(20 runs, 10 agents, for each round, for each peer-group size)

Prospect Theory
$\alpha=\beta=0.88, \lambda=2.25$

‘Traditional’ Utility model
$\alpha=\beta=0, \lambda=0$

$T\rightarrow 14.5$
$T\rightarrow ?17.0$
(UE=14)
Results (6)

Stability (propensity to change route)
(20 runs, 10 agents, for each round, for each peer-group size)

Prospect Theory
\( \alpha = \beta = 0.88, \ \lambda = 2.25 \)

‘Traditional’ Utility model
\( \alpha = \beta = 0, \ \lambda = 0 \)

\[ p: 0.5 \rightarrow 0.1 \]

\[ p: 0.5 \rightarrow ?0.5 \]
Discussion (1)

• Can the SOCIAL/DYNAMIC RP mechanism (as described in the numeric example) be associated with learning?

  **Prospect Theory**  
  **‘Traditional’ Utility model**

- Convergence of **perceived RP** towards a specific value

- Convergence of route choices and **travel times** towards a user equilibrium

- **Stability** - propensity of users to change route is to be reduced over time (less fluctuations)
Discussion (2)

• Can the SOCIAL/DYNAMIC RP mechanism (as described in the numeric example) be associated with learning?

- Convergence of perceived RP towards a specific value
  
  ![Prospect Theory](image1) ![‘Traditional’ Utility model](image2)

- Convergence of route choices and travel times towards a user equilibrium
  
  ![RP→14.5](image3) ![RP→16.0](image4)
  ![T→14.5](image5) ![T→?17.0 (UE=14)](image6)

- Stability - propensity of users to change route is to be reduced over time (less fluctuations)
  
  ![p: 0.5→0.1](image7) ![p: 0.5→?0.5](image8)
Discussion (3)

• Loss aversion - an important element of (social) learning and system stability?

  Evolutionary Psychology –
  Loss aversion can represent an optimal strategy for a person evolved to maximize his prospects for survival in environments that vary between abundance and scarcity over time (McDermott et al., 2008).

• Effect of the size of the peer-group \((n=1,2,5,10)\)?
  - No significant size effect was observed
  - Might be associated with the size and complexity of the network?
Further Research

• **Further theoretical and empirical investigation of the behavioural assumptions**
  - Calibration of model parameters
  - Validation of assumptions and results
  - Applied context
  - Social dilemmas
  - Large and complex networks

• **Application areas**
  - Modelling
  - Behavioural change
  - Connected vehicles
  - Social Networks
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Two Paradigms of Human Behaviour

The Rational Man
Homo Economicus
How people ‘should’ behave?

Bounded Rationality
Homo Psycholigicus
How people do (might) behave?

Behavioural Economics study the effects of social, cognitive, and emotional factors on economic decisions
“Cognitive Anomalies”  
(McFadden, 1999)

<table>
<thead>
<tr>
<th>Effect</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>CONTEXT</strong></td>
<td></td>
</tr>
<tr>
<td>Anchoring</td>
<td>Judgments are influenced by quantitative cues contained in the statement of the decision task</td>
</tr>
<tr>
<td>Context</td>
<td>History and presentation of the decision task influence perception and motivation</td>
</tr>
<tr>
<td>Framing</td>
<td>Equivalent lotteries, presented differently, are evaluated differently</td>
</tr>
<tr>
<td>Prominence</td>
<td>The format in which a decision task is stated influences the weight given to different aspects</td>
</tr>
<tr>
<td>Salience</td>
<td>Subjects are inconsistent in selecting and weighting the information judged salient to a decision task</td>
</tr>
<tr>
<td><strong>REFERENCE POINT</strong></td>
<td></td>
</tr>
<tr>
<td>Asymmetry</td>
<td>Subjects show risk aversion for gains, risk preference for losses, and weigh losses more heavily</td>
</tr>
<tr>
<td>Reference point</td>
<td>Choices are evaluated in terms of changes from an endowment or status quo point</td>
</tr>
<tr>
<td>Status Quo Endowment</td>
<td>Current status and history are favored relative to alternatives not experienced</td>
</tr>
<tr>
<td><strong>AVAILABILITY</strong></td>
<td></td>
</tr>
<tr>
<td>Availability</td>
<td>Responses rely too heavily on readily retrieved information, and too little on background information</td>
</tr>
<tr>
<td>Certainty</td>
<td>Sure outcomes are given more weight than uncertain outcomes</td>
</tr>
<tr>
<td>Focal</td>
<td>Quantitative information is retrieved or reported categorically</td>
</tr>
<tr>
<td>Isolation</td>
<td>The elements of a multiple-part or multi-stage lottery are evaluated separately</td>
</tr>
<tr>
<td>Primacy and Recency</td>
<td>Initial and recently experienced events are the most easily recalled</td>
</tr>
<tr>
<td>Regression</td>
<td>Idiosyncratic causes are attached to past fluctuations, and regression to the mean is underestimated</td>
</tr>
<tr>
<td>Representativeness</td>
<td>High conditional probabilities induce overestimates of unconditional probabilities</td>
</tr>
<tr>
<td>Segregation</td>
<td>Lotteries are decomposed into a sure outcome and a gamble relative to this sure outcome</td>
</tr>
<tr>
<td><strong>SUPERSTITION</strong></td>
<td></td>
</tr>
<tr>
<td>Credulity</td>
<td>Evidence that supports patterns and causal explanations for coincidences is accepted too readily</td>
</tr>
<tr>
<td>Disjunctive</td>
<td>Consumers fail to reason through or accept the logical consequences of actions</td>
</tr>
<tr>
<td>Superstition</td>
<td>Causal structures are attached to coincidences, and “quasi-magical” powers to opponents</td>
</tr>
<tr>
<td>Suspicion</td>
<td>Consumers mistrust offers and question the motives of opponents, particularly in unfamiliar situations</td>
</tr>
<tr>
<td><strong>PROCESS</strong></td>
<td></td>
</tr>
<tr>
<td>Rule-Driven</td>
<td>Behavior is guided by principles, analogies, and exemplars rather than utilitarian calculus</td>
</tr>
<tr>
<td>Process</td>
<td>Evaluation of outcomes is sensitive to process and change</td>
</tr>
<tr>
<td>Temporal</td>
<td>Time discounting is temporally inconsistent, with short delays discounted too sharply relative to long delays</td>
</tr>
<tr>
<td><strong>PROJECTION</strong></td>
<td></td>
</tr>
<tr>
<td>Misrepresentation</td>
<td>Subjects may misrepresent judgments for real or perceived strategic advantage</td>
</tr>
<tr>
<td>Projection</td>
<td>Judgments are altered to reinforce internally or project to others a self-image</td>
</tr>
</tbody>
</table>
Expected Utility Theory (EUT)
(Von Neuman & Morgenstern, 1944)

Outcomes $(x_1, \ldots, x_n)$

Utilities $(u(x_1), \ldots, u(x_n))$

Probabilities $(p_1, \ldots, p_n)$

Expected Utilities

\[ \text{Expected Utility: } p_1 u(x_1) + p_2 u(x_2) + \ldots \]
Prospect Theory
a descriptive model of decision making under risk and uncertainty

Kahneman & Tversky (1979) Prospect Theory
Tversky & Kahneman (1992) Cumulative Prospect Theory

Prospect:

\[ \pi(p_1)v(x_1) + \pi(p_2)v(x_2) + \ldots \]

Outcomes \((x_1, \ldots, x_n)\)

Values \((v(x_1), \ldots, v(x_n))\)

Probabilities \((p_1, \ldots, p_n)\)

Weights \((\pi(p_1), \ldots, \pi(p_n))\)

Prospects

Weighting function

Value function
Prospect’s Theory Weighting Function

\[ w^+(p) = \frac{p^\gamma}{[(p^\gamma + (1-p)^\gamma]^{1/\gamma}} \]

\[ w^-(p) = \frac{p^\delta}{[(p^\delta + (1-p)^\delta]^{1/\delta}} \]

- Risk aversion when outcomes are framed as gains
- Risk seeking when outcomes are framed as losses

Stated Probability: \( P \)

- Underweighting of high probabilities
- Overweighting of small probabilities
CPT formulation

Tversky & Kahneman (1992)

\[ \text{CWV} = v(f^+) + v(f^-) \]

\[ v(f^+) = \sum_{i=0}^{n} \pi^+_i v(x_i) \]

\[ v(f^-) = \sum_{i=-m}^{0} \pi^-_i v(x_i) \]

\[ \pi^+_n = w^+(p_n) \]

\[ \pi^-_{-m} = w^-(p_{-m}) \]

\[ \pi^+_i = w^+(p_i + \ldots + p_n) - w^+(p_{i+1} + \ldots + p_n) \quad 0 \leq i \leq n-1 \]

\[ \pi^-_i = w^-(p_{-m} + \ldots + p_i) - w^-(p_{-m} + \ldots + p_{i-1}) \quad 1-m \leq i \leq 0 \]

\[ \pi^+(0) = w^-(0) = 0 \]

\[ \pi^+(1) = w^-(1) = 1 \]
Setting a value to the reference point

- Money can be pooled / stored
- Time can not be stored

- 0 is the common reference point for gains / losses
- What is the reference point in the context of travel choice?
Violations of EUT in route-choice situations

preferences revealed in some problems violate the EUT assumptions (Avineri & Prashker, 2004)

These violations of EUT assumptions are not based on a specific shape of the utility function

a particular pattern of risk attitude is captured: risk aversion when lotteries are framed as gains risk seeking when lotteries are framed as losses

Robustness of Kahneman’s & Tversky’s results
Applications of Prospect Theory to Travel Choice Modelling

- Viti et al. (2005)
- Han et al. (2005)
- Chen & Mahmassani (2006)
- Zhao & Zhang (2006)
- Dell’Orco et al. (2007)
- Polak, Hess, & Liu (2008)
- Schwanen & Ettema (2009)
- van de Kaa (2010)
- EJTIR Special Issue (2010)
- Gao, Frejinger, Ben-Akiva (2010)
- ...

- SP
- Small databases
- Static situations
- Not all features of PT incorporated/explored