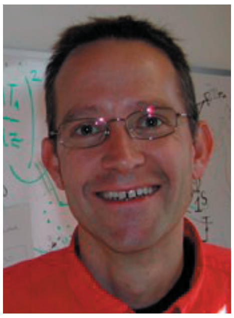
Work minimisation in rock and snow avalanches: The competition between random kinetic energy and heat

Wechselwirkung von Fluktuation und Dissipation in granularen Lawinen: Die Rolle der freien kinetischen Energie in schnellen granularen Strömungen



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The extremely long runout distances of snow and rock avalanches implies low friction. Why this friction is so low, however, has never been fully explained. In the following we show that low friction is the result of a mechanical regulation involving the production of random kinetic energy at the basal sliding surface of the avalanche. Observations show that the avalanche flow height profile can be divided into three layers: the slip-volume, the fluidized layer and flow bulk. The thickness of these layers is determined by the avalanche itself, primarily by the slip-velocity – the velocity at the interface between the slip volume and fluidized layer. An energy analysis reveals that both heat and random kinetic energy of the granules are produced in the slip volume. This random energy diffuses into the adjoining layer and creates the fluidized layer, thereby producing an apparent reduction in the flow friction. The penetration depth of the diffused random kinetic energy defines the depth of the fluidized layer. The partitioning of energy in the boundary layer into heat and kinetic energy (which is later dissipated in the fluidized layer) represents a competition between two mechanical processes that the avalanche can self-regulate to minimize frictional resistance.

Die erstaunlich langen Auslaufstrecken bei Schneelawinen wird auf einen kleinen Reibungskoeffizienten abgeschoben. Wie dieser allerdings zustande kommt, wird nicht erklärt. In der vorliegenden Arbeit wird der Grund in einer Art Selbstregulierung gesucht. Dazu wird die Fliesshöhe in drei Schichten unterteilt – Die Gleitschicht (slip volume), die Fliesschicht (fluidized layer) und den festen Block (bulk). Die Aufteilung in Fliessschicht und Block ist historisch bedingt. Die Dicke dieser Schichten wird von der Lawinen selber bestimmt, und zwar im wesentlichen durch die Gleitgeschwindigkeit (slip velocity), das heisst die Geschwindigkeit der Grenzschicht zwischen Gleit- und Fliessschicht. Eine Energiebetrachtung zeigt, dass im Gleitvolumen nicht nur Wärme erzeugt wird, sondern auch freie (random) kinetische Energie erzeugt wird, in Form von bewegten Teilchen die sich in beliebiger Richtung bewegen. Diese Energie (Bewegung) diffundiert in die angrenzende Schicht und erzeugt damit die Fliessschicht. Sie bewirkt eine scheinbare Erniedrigung des Reibungskoeffizienten. Die Eindringtiefe dieser diffundierenden Energie in den festen Block definiert die Dicke der Fliesschicht. Die Aufteilung der Energie in Gleitschicht in Wärme und kinetische Energie, welche erst in der anschliessenden Fliesschicht dissipiert wird, führt zu einer Konkurrenzsituation, welche die Lawine ausnützt, um so wenig wie möglich Reibungswärme zu erzeugen.

Introduction

A long-standing problem in understanding the motion of large catastrophic avalanches of debris, rock or snow is finding a physical mechanism that explains the extremely low friction values that result in far-reaching, and therefore potentially dangerous, runout distances. Various mechanisms have been postulated to explain this phenomena, including heating/lubrication (Hsü, 1975), dynamic fragmentation (Davies et. al., 1999) or granular fluidization (Collins and Melosh, 2003).

In this paper we address this problem by deriving general relationships between the basal work rate and the production of random kinetic energy (granular agitation) and internal energy (heat) in rapid mass flows. These relations place thermodynamic restrictions on constitutive formulations describing the motion of rock or snow avalanches. Subsequently, they can be used to test constitutive models describing the interaction of a rapid mass flow with the basal running surface, i.e. the physical mechanisms postulated to explain the extreme mobility of large avalanches. In the following we present a constitutive formulation based on these “fluctuation-dissipation relations” and apply it to model shear stresses measured in granular and snow avalanche chutes (Fig. 1).

Figure 1: Large scale chute experiments with snow (left) and laboratory chute experiments with granular materials are used to measure basal shear stresses S and normal stresses N (middle). The chutes are instrumented with shear and normal force plates and optical flow height sensors (right).

The fluctuation-dissipation relations, however, cannot explain the low friction values alone. Therefore, we must also address the question if flow states exist in which the friction/dissipation is minimal within the given flow constraints (mass flux, slope angle, etc.). Mathematically, this is a variational problem which requires identifying competing frictional processes in both the basal shear layer and avalanche bulk. Interestingly, we find that not only do such flow states exist, but they are also remarkably stable with respect to the production of random kinetic energy in the flow system (Bartelt et al. 2005).

Fluctuation-Dissipation Relations

When an avalanche descends down a mountain slope, gravitational potential energy is transformed into translational kinetic energy and internal energy. Gravitational work raises both the speed and thermal temperature of the flowing mass. Three physical processes are responsible for the heat rise: (1) the sliding shearing at the basal plane, (2) the viscous shearing within the avalanche bulk which arises from the enduring frictional contact between granules of rock or snow and (3) the random inelastic collisions between granules (Bartelt et al. 2006). These three processes are mathematically parameterized by the avalanche slip velocity $u_0\,(x)$, the velocity distribution within the avalanche body $u(x, z)$ and the square of the random granule velocity $f(x, z)$, representing the random kinetic energy of the flow (Fig. 2). The functions $u_0\,(x)$, $u(x, z)$, and $f(x, z)$ are presently unknown; however, their values define the frictional work rates opposing gravity and thereby the internal energy distribution in the falling mass. These functions cannot be found without postulating some constitutive relations for the basal and viscous shearing and the creation, transport and destruction of random kinetic energy within the avalanche. Further, we expect the random kinetic energy $f(x, z)$ will influence the work done by the enduring frictional contacts between granules and thus the velocity distribution $u(x, z)$.

The total frictional work done by the three processes can be defined at any position x, without exactly stipulating the constitutive relations:

$$\dot W''_{nc} = \dot W''_0 + \int_0 \dot W'''_v(z)\,dz + \int_0 \dot W'''_r(z)\,dz = \dot W''_0 + \dot W''_v + \dot W''_r \quad (1)$$

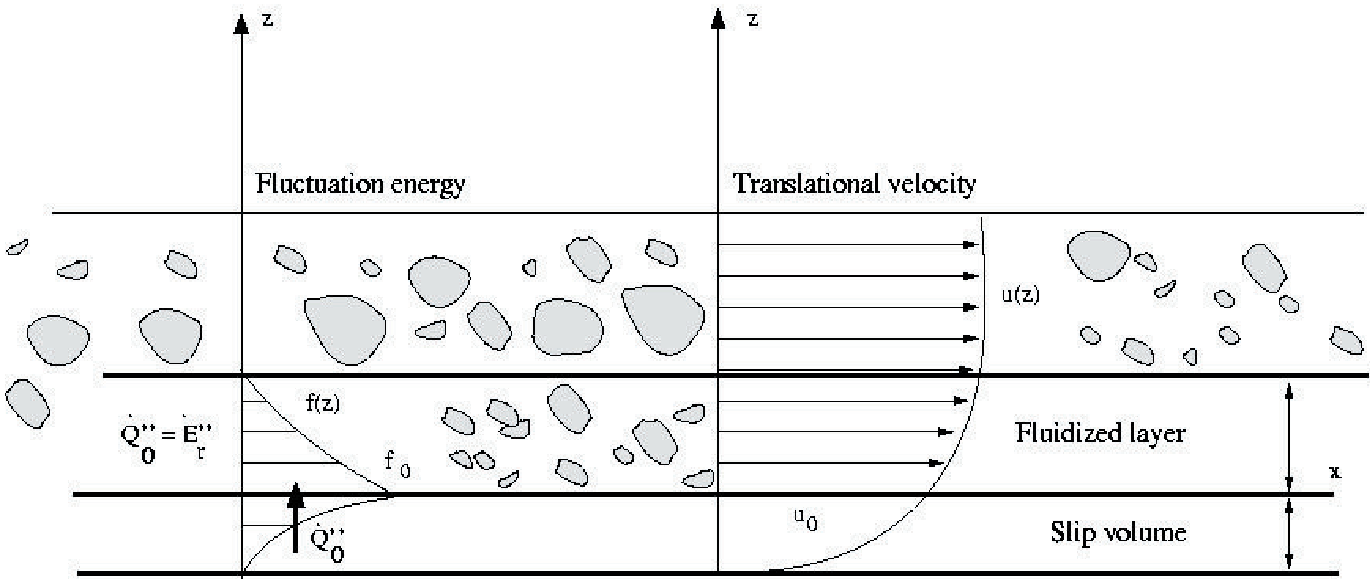

Figure 2: The avalanche profile is divided into three layers: the slip volume (boundary layer), fluidized layer and flow bulk, defined where by $f(x, z) = 0$. At the interface between the slip volume and fluidized layer we have the slip velocity $u_0\,(x)$ and injection of random kinetic energy $\dot{Q}''_0$.

where $\dot{W}''_{nc}$ the total frictional work rate of the three non-conservative processes (W/m^2); $\dot{W}''_0$ is the basal shear rate (W/m^2); $\dot{W}'''_v$ is the viscous shear rate (W/m^3) and $\dot{W}'''_r$ is the work done by the random collisions (W/m^3). A double prime subscript denotes a quantity per unit area whereas a triple prime subscript a quantity per unit volume. Integration over the avalanche flow height h is required to add the work done in the avalanche bulk to the work done at the basal shear layer. The integration projects the volumetric work rates on to the basal surface.

The total rise in internal energy $\dot{E}''$ can likewise be stated:

$$\dot{E}'' = \dot{E}''_0 + \int_0^h \dot{E}'''_v(z)dz + \int_0^h \dot{E}'''_r(z)dz = \dot{E}''_0 + \dot{E}''_v + \dot{E}''_r \quad (2)$$

where the sub- and superscripts of E have the same meaning as in Eq. 1.

In steady-state, the integrated rise in internal energy $\dot{E}''$ is in balance with the gravitational work rate $\dot{W}''_g$, or alternatively, the work done by the non-conservative forces:

$$\dot{W}''_g = \dot{E}'' = \dot{W}''_{nc}\,. \quad (3)$$

The internal energy rise due to the random inelastic collisions $\dot{E}''_r$ is certainly non-zero, for all non-zero distributions of $f(x, z)$. However, the mechanical work done, in the time-averaged mean, by the random collisions $\dot{W}''_r$ is zero, since the randomness of the collisions is defined to have no positive or negative bias with respect to the mean velocity of the avalanche. The random collisions can neither accelerate nor decelerate the flow. Therefore,

$$\dot{W}'''_r(z) = 0 \qquad \text{for all } z. \quad (4)$$

Because $\dot{E}'' = \dot{W}''_{nc}$ in steady state, we have

$$\dot{W}''_0 = \dot{E}''_0 + \int_0^h \dot{E}'''_r(z)dz \quad (5)$$

since the internal energy rise and work done by the viscous shearing is in balance

$$\int_0^h \dot{W}'''_v(z)dz = \int_0^h \dot{E}'''_v(z)dz. \quad (6)$$

Let be $\dot{Q}''_0$ defined as

$$\dot{Q}''_0 = \int_0^h \dot{E}'''_r(z)dz \quad (7)$$

then

$$\dot{W}''_0 = \dot{E}''_0 + \dot{Q}''_0\,. \quad (8)$$

The quantity $\dot{Q}''_0$ represents the flux of fluctuation (agitation) energy injected at the base of the avalanche. Eq. 8 states that the total frictional work done at the base of the avalanche $\dot{W}''_0$ can be split into two parts: Part of the basal shear work is immediately dissipated directly into heat $\dot{E}''_0$ while the remaining work produces random kinetic energy $\dot{Q}''_0$, which in steady-state is later dissipated – in the avalanche body – by inelastic collisions. The random motion of the granules represents an intermediate stage in which potential energy is briefly stored as random kinetic energy before it too is dissipated into heat. Thus, part of the basal shear work raises the thermal temperature of the system, while the remaining part, raises the fluctuation energy, also called the granular temperature. Eq. 7 is a general requirement for granular avalanche flow in steady-state; whereas Eq. 8 has much wider application because it is also valid for non-steady flows.

Constitutive Model for Basal Shearing

A constitutive model for basal shear S_0 that satisfies the fluctuation-dissipation relation is

$$\dot{W}''_0 = \dot{E}''_0 + \dot{Q}''_0 \quad \text{is}$$

$$S_0 = \mu(\dot{Q}''_0)N_0 \quad (9)$$

where $\mu(\dot{Q}''_0)$ is the basal friction coefficient, which depends on the fluctuation energy input $\dot{Q}''_0$; N_0 is the normal stress acting on the basal shear plane. A relation for $\mu(\dot{Q}''_0)$ which agrees well with shear and normal stress measurements of granular avalanches captured in simple chute experiments (Figs. 3 and 4) is

$$\mu(\dot{Q}''_0) = \mu_s\left[1 - \frac{\dot{Q}''_0}{\dot{Q}''_\infty}\right] + \mu_d\frac{\dot{Q}''_0}{\dot{Q}''_\infty} = \mu_s + \left(\mu_d - \mu_s\right)\cdot \quad (10)$$

where μ_s is the "static" coefficient of friction; μ_d is the "dynamic" coefficient of friction, $\mu_d > \mu_s$. The quantity $\dot{Q}''_\infty$ represents the work rate in a fully dynamic or fluidized regime

$$\dot{Q}''_\infty = \mu_d N_0 u_0 > \dot{Q}''_0\,. \quad (11)$$

An experimentally determined relation for $\dot{Q}''_0$ is

$$\dot{Q}''_0 = \dot{Q}''_\infty\left[\frac{u_0}{B + u_0}\right] \quad (12)$$

where B is a function of h and u_0 which defines how the static flow changes to the dynamic flow state, given by μ_d. That is, the relation theoretically fits the following limit conditions. When

$$u_0 >> B \qquad \text{then} \quad \dot{Q}''_0 \rightarrow \dot{Q}''_\infty \quad (13)$$

and

$$u_0 >> B \qquad \text{then} \quad \dot{Q}''_0 \rightarrow 0\,. \quad (14)$$

The function B depends on both the flow material and the roughness of the flow surface. Note that when

$$\dot{Q}''_0 \rightarrow \dot{Q}''_\infty \qquad \text{then} \quad \frac{S_0}{N_0} \rightarrow \mu_d \quad (15)$$

and

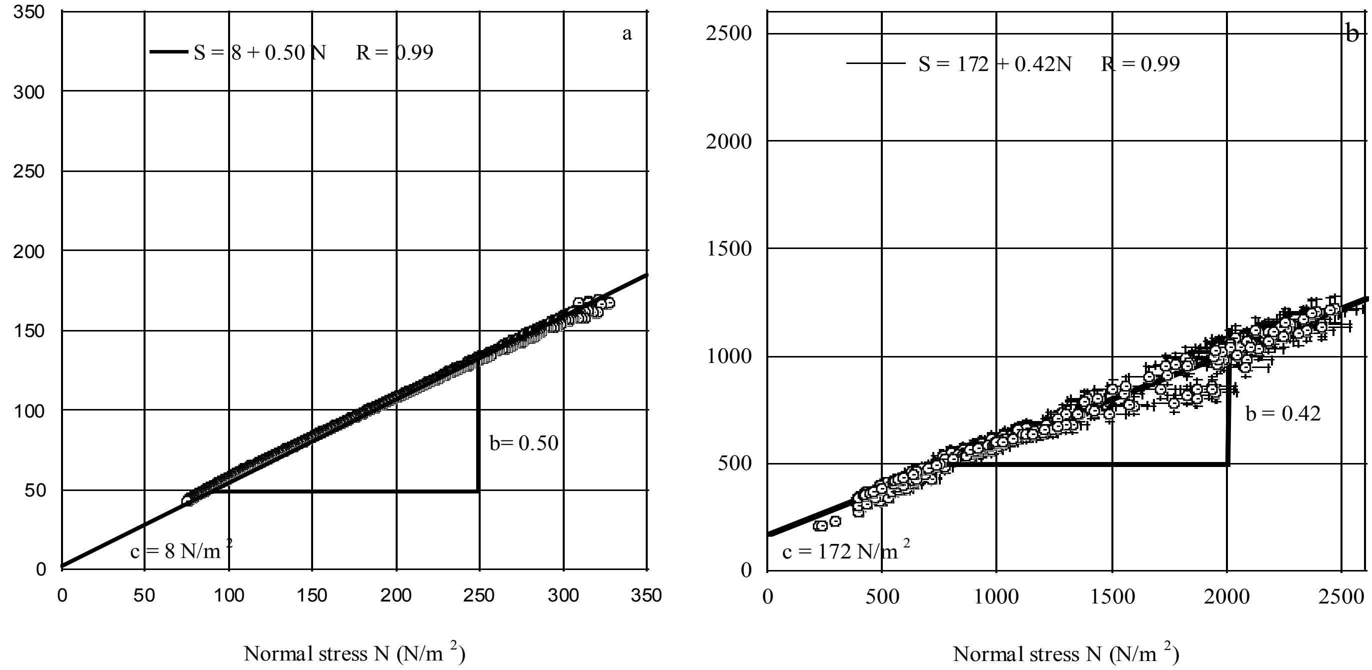

Figure 3: Measured shear S and normal N forces for a: granular experiments and b snow chute experiments. A simple Mohr-Coulomb relation appears adequate Eq. 9. See Figure 3, however, for non-linear relation between S/N and Fr number.

and

$$\dot{Q}_0'' \to 0 \qquad \text{then} \qquad \frac{S_0}{N_0} \to \mu_s \,. \tag{16}$$

Moreover, this general constitutive relation fulfils the limit requirements that when the fluctuation energy input $\dot{Q}_0''$ decreases to zero, basal friction is given by the static friction coefficient μ_s whereas when the fluctuation energy input $\dot{Q}_0''$ reaches the dynamic limit $\dot{Q}_\infty''$, basal friction is defined by the dynamic value μ_d.

The internal energy production (heat) $\dot{E}_0''$ at the basal layer is

$$\dot{E}_0'' = \mu_s\left[1-\frac{\dot{Q}_0''}{\dot{Q}_\infty''}\right]N_0u_0 = \mu_s N_0 u_0 - \frac{\mu_s}{\mu_d}\dot{Q}_0'' \tag{17}$$

that is, in the dynamic limit

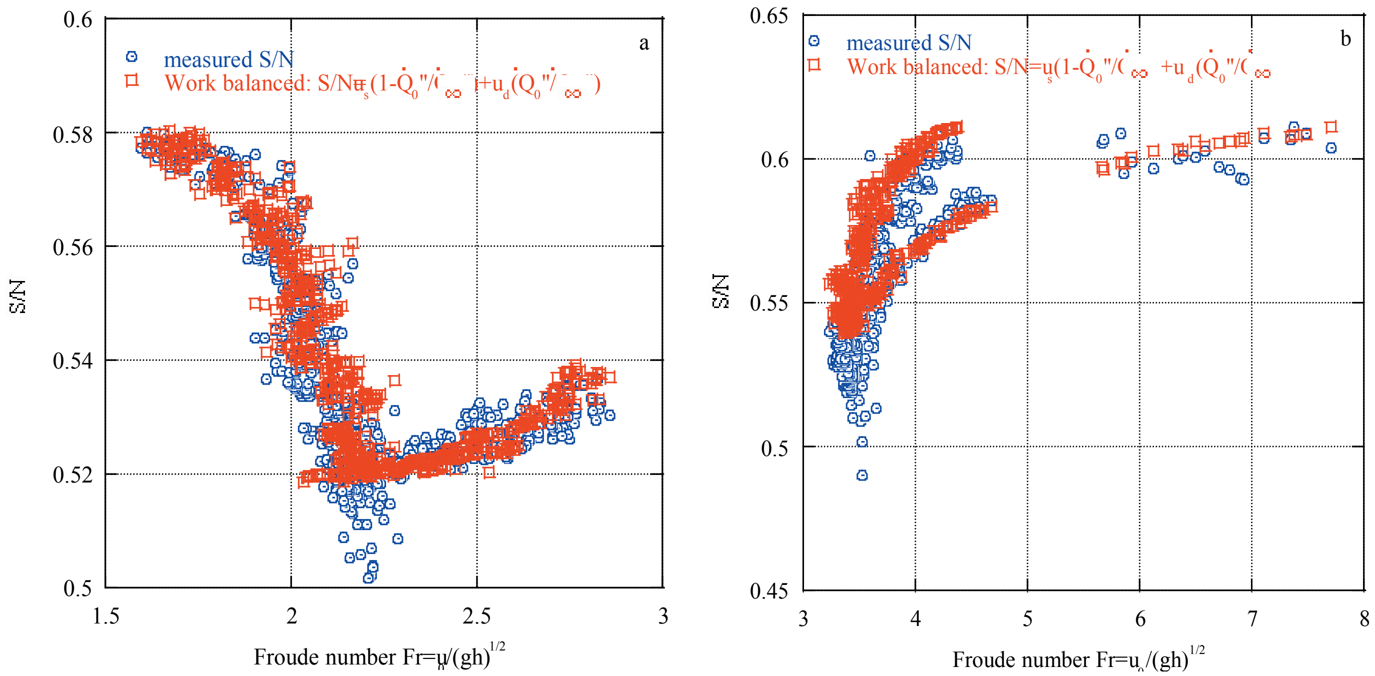

Figure 4: Plot of measured S/N ratio as a function of Froude number Fr for two granular experiments a: chute inclination angle 24°; b: chute inclination angle 26° (Figure 1). Fit of Eq. 10 to experimental measurements. Both the head and tail of the avalanche are accurately represented.

$$\dot{Q}''_0 \rightarrow \dot{Q}''_\infty \qquad \text{then} \quad \dot{E}''_0 \rightarrow 0\,. \tag{18}$$

An alternative constitutive formulation is the Voellmy-Salm friction law (Norem et al., 1987; Salm, 1993), which has often been used for snow avalanches (Bartelt et al., 1999), with

$$S_0 = \mu N_0 + s u_0^2 \tag{19}$$

or

$$\dot{W}''_0 = \dot{E}''_0 + \dot{Q}''_0 = \mu N_0 u_0 + s u_0^3\,. \tag{20}$$

The production of random kinetic energy is simply given by $\dot{Q}''_0 = su_0^3$, where s is a "Chezy" turbulence-like parameter (see Salm (1993).) This law can be applied to simulate the measured basal shear stresses in chute experiments; however, it underestimates the basal friction at the tail of the flow (see Fig. 5).

The Fluidized Layer

The random kinetic energy that is produced in the slip volume "fluidizes" all or part of avalanche core. The fluidized layer height (Fig. 2) depends on how strongly the random kinetic energy is dissipated by the inter-granular collisions. That is, it depends on the inelasticity of the granules. The transport of fluctuation energy is therefore described by a simple diffusion equation with the accompanying boundary condition governing the injection of fluctuation energy from the slip volume:

$$k\frac{\partial^2 f(z)}{\partial z^2} = \frac{k}{\alpha^2} f(z) \qquad \text{with} \quad -k\left.\frac{\partial f}{\partial z}\right|_{z=0} = \dot{Q}''_0 \tag{21}$$

where k is the conduction coefficient governing the diffusion of fluctuation energy in the fluidized layer and α accounts for the destruction of fluctuation energy by inelastic collisions. The solution to this equation is

$$f(z) = f_0 e^{\left(-\frac{z}{\alpha}\right)} \tag{22}$$

where f_0 is the fluctuation energy at the base of the flow $z = 0$, which is determined from the boundary condition

$$f_0 = \dot{Q}''_\infty \left[\frac{u_0}{B + u_0}\right]\left[\frac{\alpha}{k}\right] \tag{23}$$

The fluidized layer height is the height at which

$$\int_0^{h_f} f(z)\,dz = 0\,. \tag{24}$$

The fluctuation energy distribution $f(z)$ influences the translation velocity distribution in the fluidized layer $u(z)$. Two possible interactions are:

- The shear viscosity v decreases in the fluidized layer as a function of $f(z)$. Moreover, the shear stress distribution is

$$S(z) = v(f(z))\frac{\partial u(z)}{\partial z} \tag{25}$$

- The fluctuation energy produces a dispersive pressure $P(z)$ which acts against the overburden stress $N(z)$

$$S(z) = b\left[N(z) - P(z)\right] + v\frac{\partial u(z)}{\partial z} \tag{26}$$

Here experimental evidence is failing to describe the interaction between $f(z)$ and $u(z)$. Bartelt et al. (2006) applied

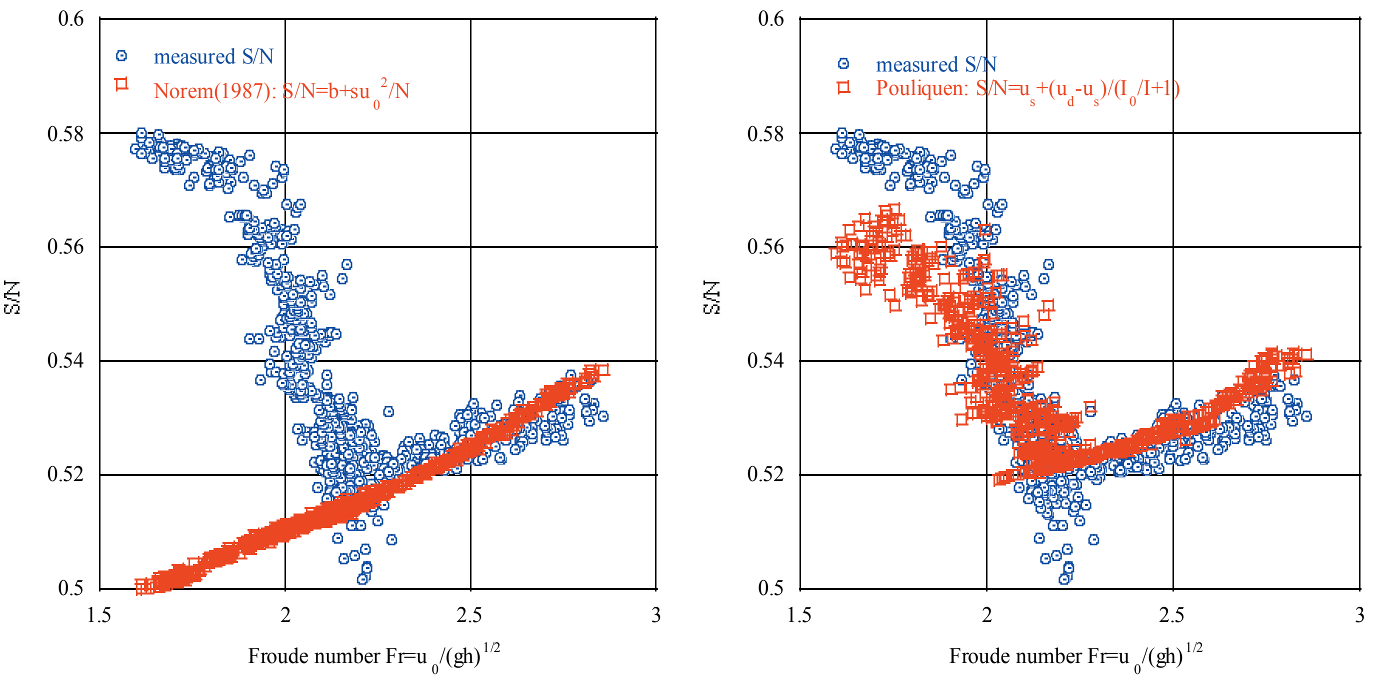

Figure 5: Right: Fit of experimental to Voellmy-fluid type relation (Eq. 20) used by Norem et al. (1987). The model can match shear stresses at the avalanche front, but not at the avalanche tail. Left: Fit of the same experiment with the constitutive relation proposed by Job et al. (2006).

the second constitutive formulation and found that they could accurately describe the plug-like velocity distributions in wet snow avalanches. Energy dissipation decreases in the fluidized layer, with increasing supply of fluctuation energy $\dot{Q}''_0$ in both constitutive formulations. Finally, it can be shown that the interaction between the viscous shearing and the random fluctuations is reciprocal in the second formulation, fulfilling the thermodynamic constraint that the interaction between two dissipative processes is defined by a single and therefore unique heat producing mechanism (see Bartelt et al., 2006).

Mechanical Work/Friction Minimization

Although the work balanced constitutive formulation accurately fits the experimentally measured shear stress (Fig. 4), it does not explain the extreme mobility of large mass flows. In fact, the theory suggest that with increasing velocity and mass, an avalanche will be governed by the dynamic coefficient of friction μ_d which is always larger than μ_s.

To explain the reduced friction we must investigate the total, non-conservative work (Eq. 1) $\dot{W}''_{nc}$ which consumes at any instant only part of the gravitational work rate $\dot{W}''_g$. The remaining part of the gravitational work raises the translational kinetic energy of the avalanche. Obviously, the smaller the frictional work rate $\dot{W}''_{nc}$, the higher the speed of the movement.

In order to find the minimal frictional work rate, we return to Eq. 1, which divides the work rate into two parts:

$$\dot{W}''_{nc} = \dot{W}''_0 + \dot{W}''_v \ . \qquad (27)$$

The work done in the basal slip volume $\dot{W}''_0$ *increases* as a function of the slip velocity u_0 (Fig. 6) since

$$\dot{W}''_0 = S_0 u_0 = \mu(\dot{Q}''_0) N_0 u_0 = \mu_s N_0 u_0 + (\mu_d - \mu_s) N_0 \frac{u_0^2}{B + u_0} \qquad (28)$$

However, the viscous shear work done in the fluidized layer $\dot{W}''_v$

$$\dot{W}''_v = \int_0^{h_f} S(z) \frac{\partial u(z)}{\partial z} dz \approx \nu \frac{[u - u_0]^2}{h_f} \qquad (29)$$

decreases as a function of the slip velocity u_0 (Bartelt et al., 2005). As u_0 approaches the mean velocity of the avalanche u, shear deformations, and therefore the frictional work rate, will decrease to zero (Eq. 29, Fig. 6). This analysis shows that with respect to u_0, basal shearing in the slip volume and viscous shearing in the fluidized layer are competing processes. When the one increases, the other will decrease and vice-versa. Therefore, there exists a u_0 at which the sum of the two frictional work rates is minimal (Fig. 6).

This is an intriguing result because it suggests that if avalanches can find a u_0 that minimizes the frictional work rate, they will be travelling as fast, and therefore, as far as they possibly can within the governing constraints such as the mass flow and slope angle. Interestingly, the parabolic shape of the total frictional work curve (Fig. 6)

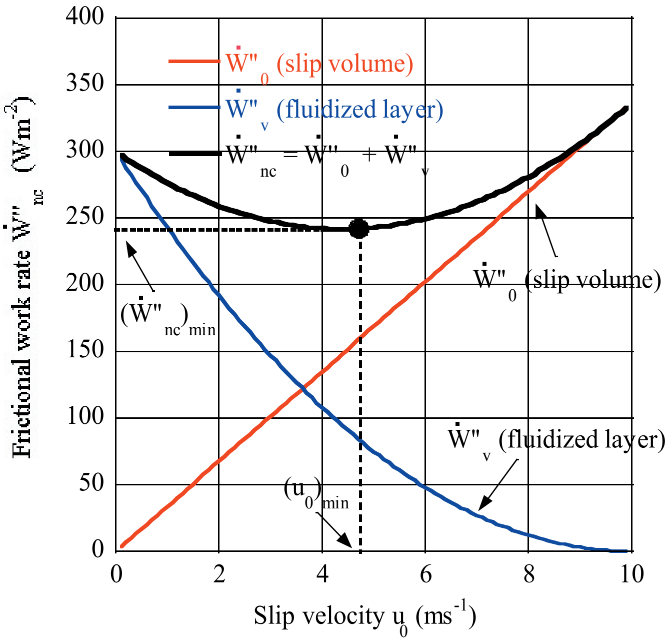

Figure 6. The total frictional work $\dot{W}''_{nc}$ is composed of two parts: the basal shear work rate in the slip volume $\dot{W}''_0$ and viscous shear work rate in the fluidized layer $\dot{W}''_v$. There exists a slip velocity u_0 that minimizes the sum. $\dot{W}''_0$ and $\dot{W}''_v$ are competing dissipating processes.

indicates that the flow system is stable with respect to all perturbations in slip velocity u_0 in the sense of Lyapunov (Bartelt et al., 2005). Thus, once in this minimal flow state, any perturbation of u_0 at the slip-volume/fluidized layer interface, will return to the state of least frictional work. This result is only valid for cases where the fluctuation energy production $\dot{Q}''_0$ is consumed entirely within the fluidized layer $h_f < h$.

Conclusions

The idea that the granules in a large avalanche have some random kinetic energy – as well as their translational kinetic energy – is well accepted. However, important questions remain: where is the source of this granular energy, how strong is it and how and where is it dissipated? Further, how does the random kinetic energy interact with other frictional mechanisms? The work-energy analysis suggests some answers.

The only plausible source of the random kinetic energy is at the bottom of the avalanche. Furthermore, this source must have a counterpart, a sink – especially when the avalanche has reached a terminal velocity and steady state. This result leads us to assume two volumes: the source in the slip volume, and the sink in the fluidized layer. These two volumes are created by the movement of the avalanche, or more precisely by the slip velocity, defined as the velocity at the boundary between slip and fluidized layer, which is of course related to the "avalanche speed", or the front velocity of the flow. The random kinetic energy produced in the slip layer diffuses into the bulk and forms the fluidized layer. No fluidized layer can exist when there is no random kinetic energy produced in the slip layer. This random kinetic energy in the fluidized layer reduces the frictional forces acting on the avalanche. The two-volume model has already proved helpful in constructing a constitutive relation that accurately models the measurements of basal shear stresses in laboratory experiments and snow chute experiments.

However, the diffusion of random kinetic energy in the fluidized layer also causes a reduction in the slip velocity, which, in turn, throttles the production of random kinetic energy. The non-linear interaction between the slip volume and fluidized layer can be viewed as competing mechanisms.

Interestingly, the competition to consume gravitational work at the base of the avalanche implies that there is a slip velocity at which the dissipated work, or the heat produced, is minimal. All perturbations in slip velocity caused by changes in boundary conditions, such as slope angle and roughness, will be driven to assume this minimal value.

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