Mechanically Efficient Cellular Microstructures in Plants

Lorna J. Gibson

Materials Science & Engineering

MIT

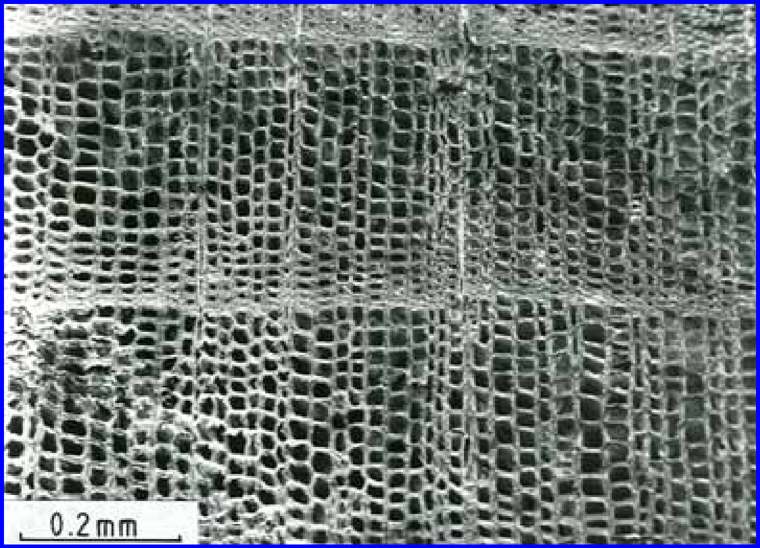
Introduction

- Plants are typically loaded in bending by wind and in compression by self-weight
- Minimizing mass reduces metabolic cost to grow material
- Examine strategies used in plants to reduce mass

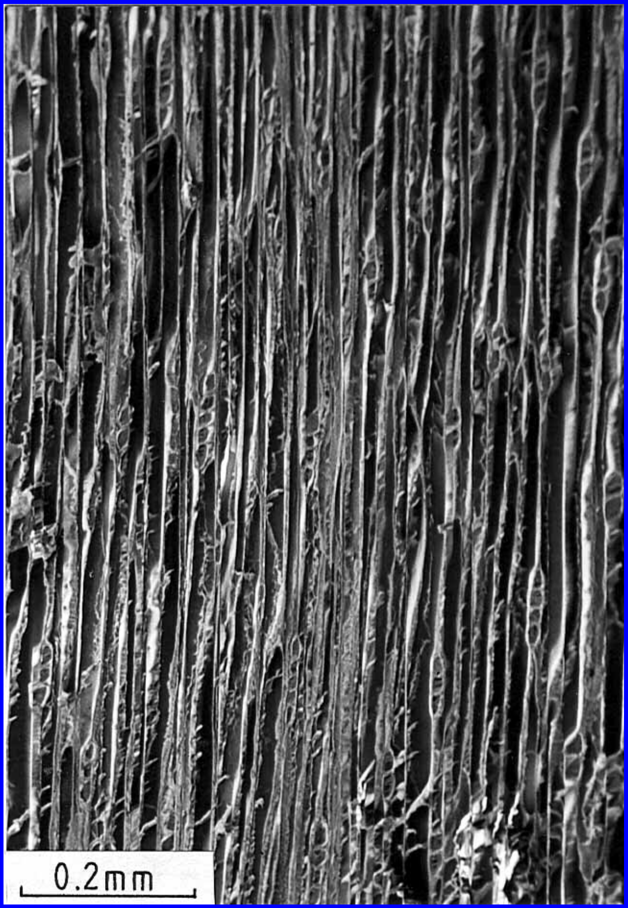
Introduction

- *Wood*: Uniform honeycomb-like structure
- *Palm stem*: Radial density gradient
- *Plant stem*: Cylindrical shell with compliant core
- *Monocotyledon leaves*: Sandwich structures

Wood:
Honeycomb-Like Microstructure



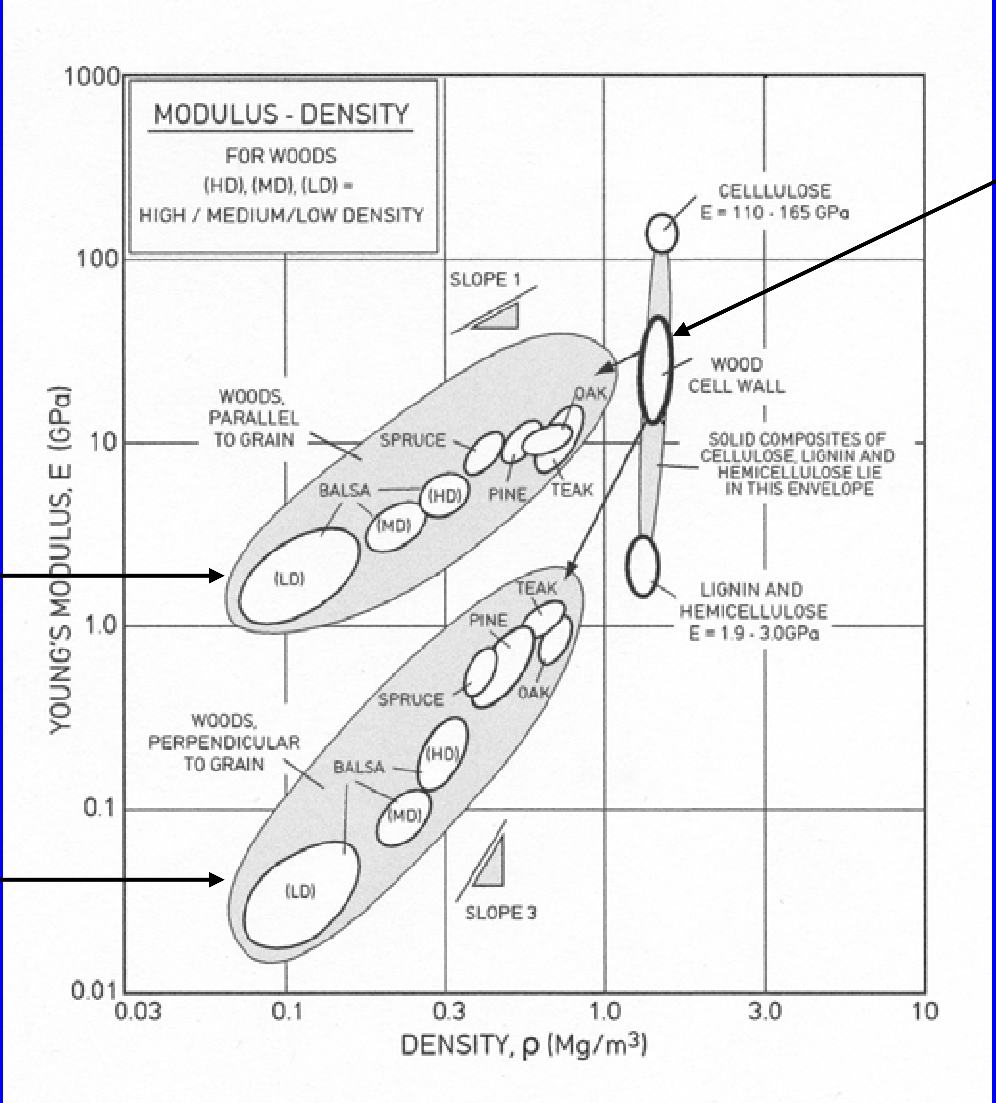
Cedar



Wood: Honeycomb Models

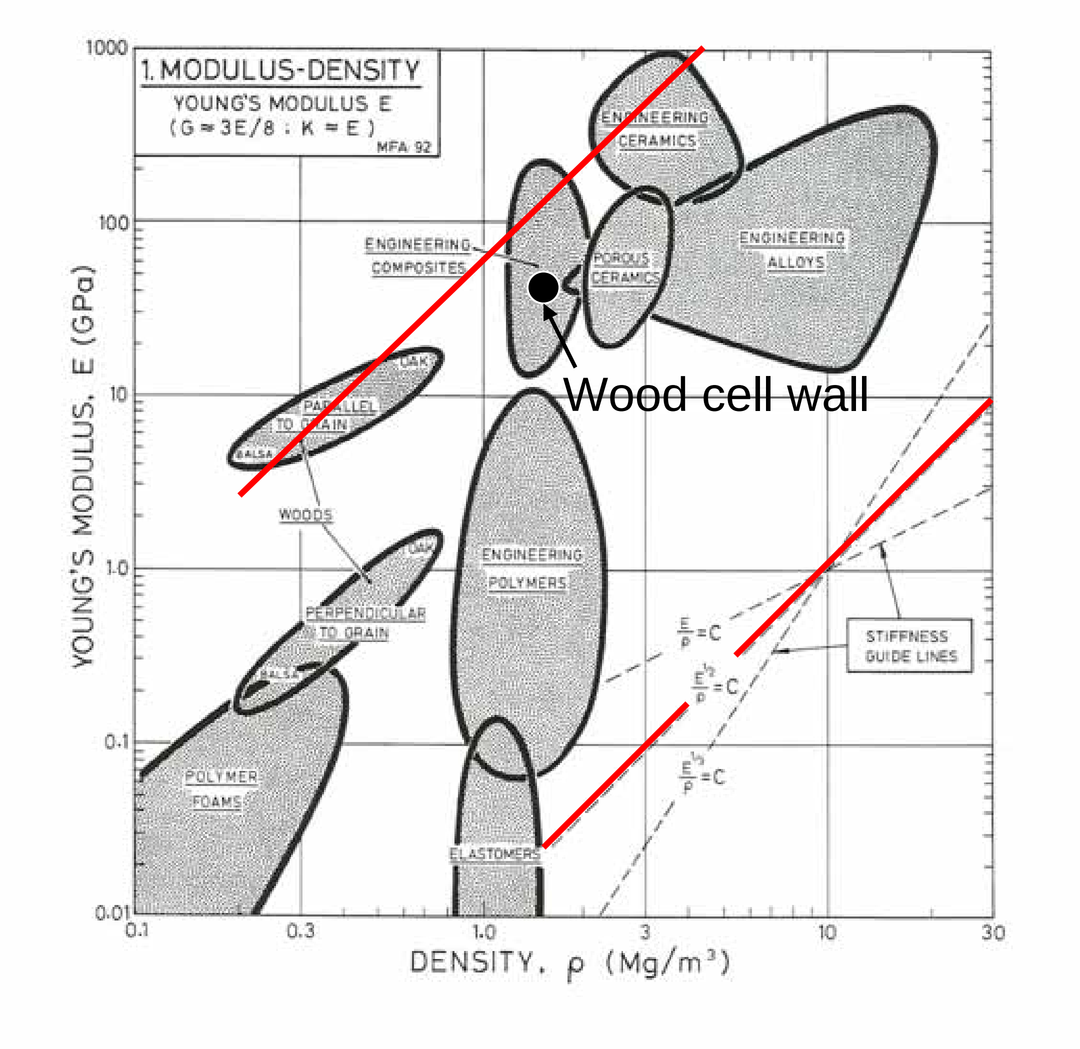
$$\frac{E^*}{E_{s\,along}} = \frac{\rho^*}{\rho_s}$$

$$\frac{E^*}{E_{s\,across}} = \left(\frac{\rho^*}{\rho_s}\right)^3$$



Cell wall:
Fiber
Composite
Model

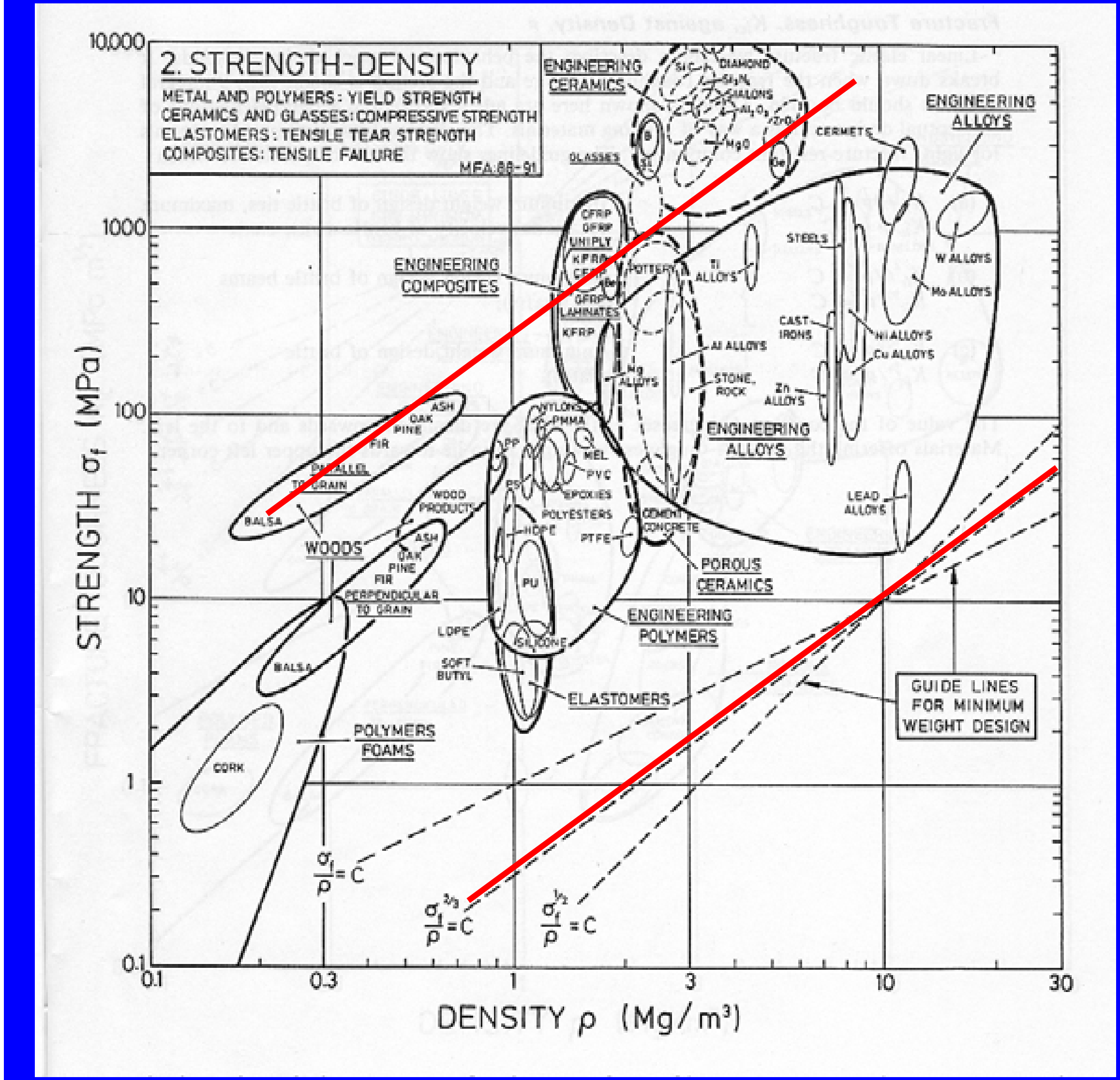
Wood in Bending: $E^{1/2}/\rho$



$$\frac{\left(E^*\right)^{1/2}}{\rho^*} = \frac{\left(E_s\right)^{1/2}}{\rho_s}\left(\frac{\rho_s}{\rho^*}\right)^{1/2}$$

Stiffness performance index for wood in bending is similar to that for best engineering composites

Wood in Bending: $\sigma_f^{2/3}/\rho$



$$\frac{\left(\sigma_f^*\right)^{2/3}}{\rho^*} = \frac{\left(\sigma_{ys}\right)^{2/3}}{\rho_s}\left(\frac{\rho_s}{\rho^*}\right)^{1/3}$$

Strength performance index for wood in bending is similar to that for best engng composites

Wood

- Tree in bending loaded as cantilever
- Radius decreases with distance away from the ground
- Further increases mechanical performance of the tree
- E = constant, r = r (z)

Palm Stem:
Radial Density Gradient

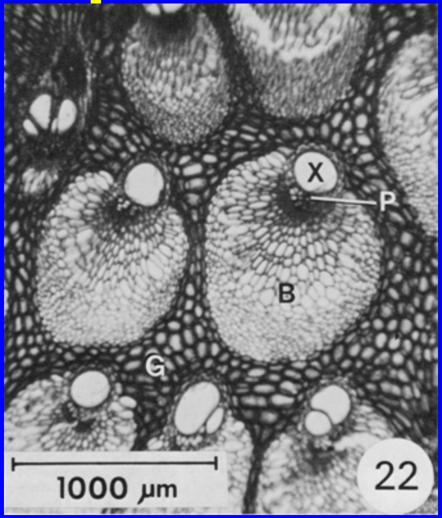
(Also Bamboo)

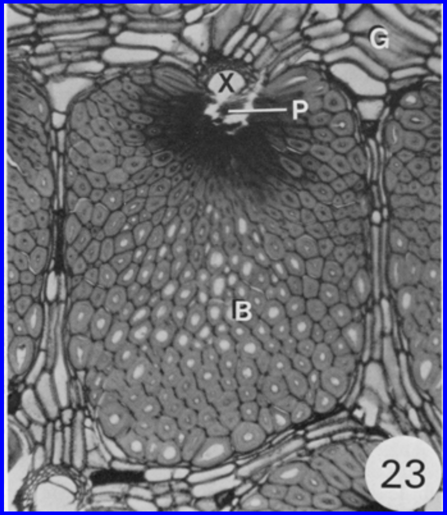
Palm Stem: A Different Strategy

- Stem has constant diameter: r = constant
- As palm grows taller, it increases the density of the material towards its periphery
- Cell wall thickness increases towards periphery of stem and towards the base of the stem E = E (r, z)

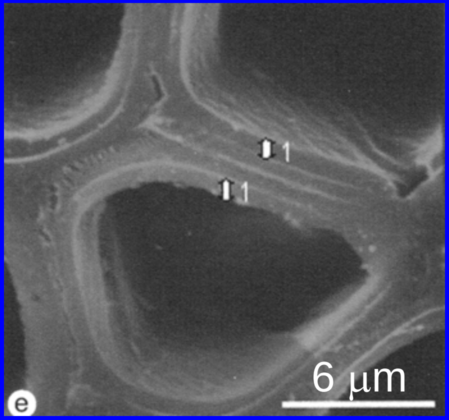
Coconut Palm
http://en.wikipedia.org/wiki/
Image:Palmtree_Curacao.jpg

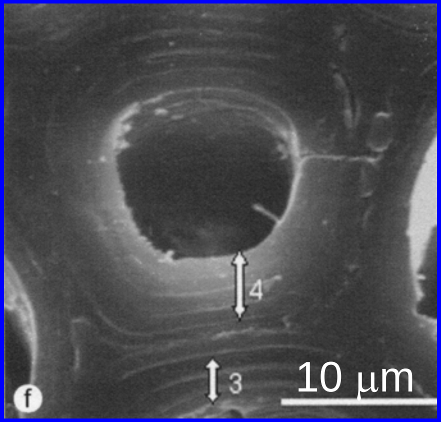
Palm: Microstructure of Peripheral Stem Tissue





Rich, 1987



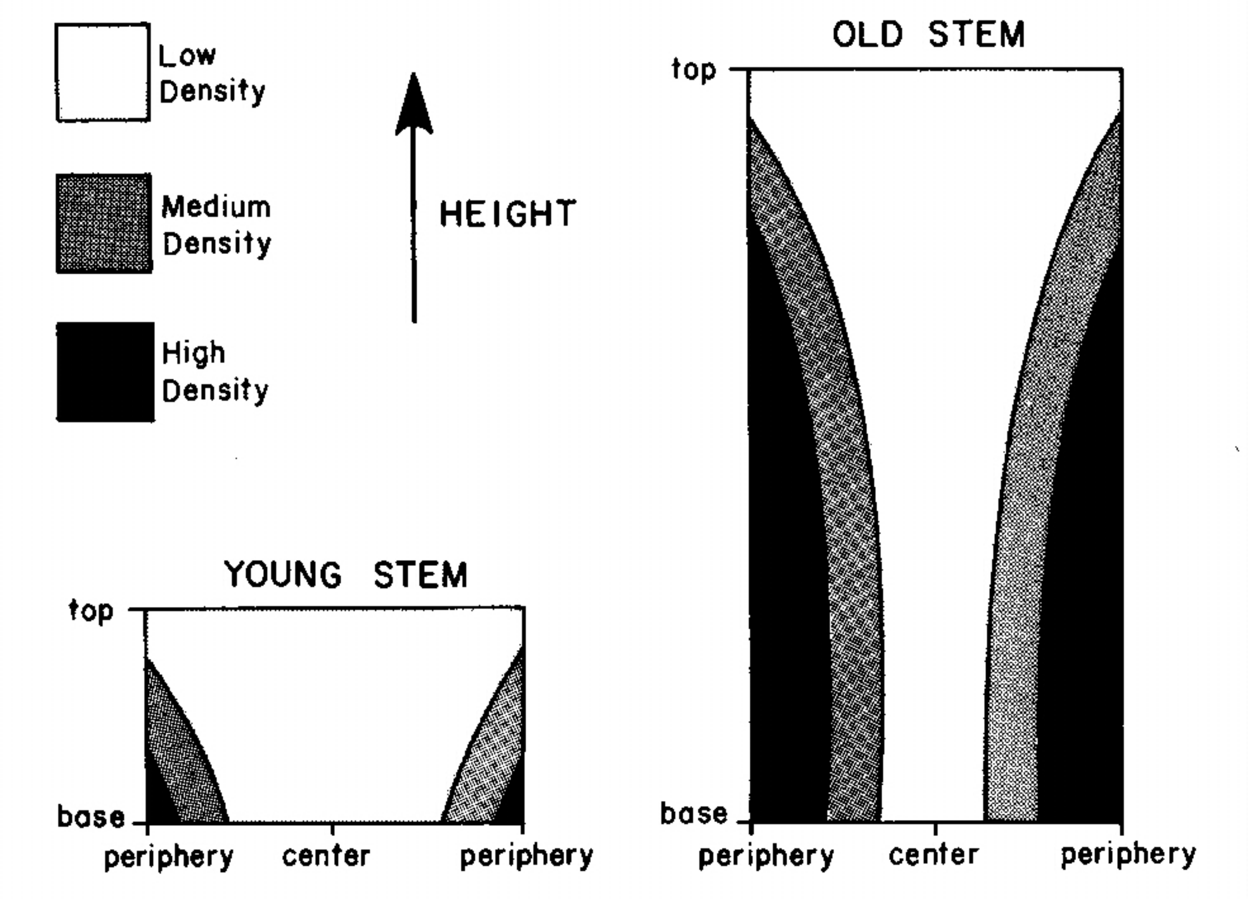


Kuo-Huang et al., 2004

Young

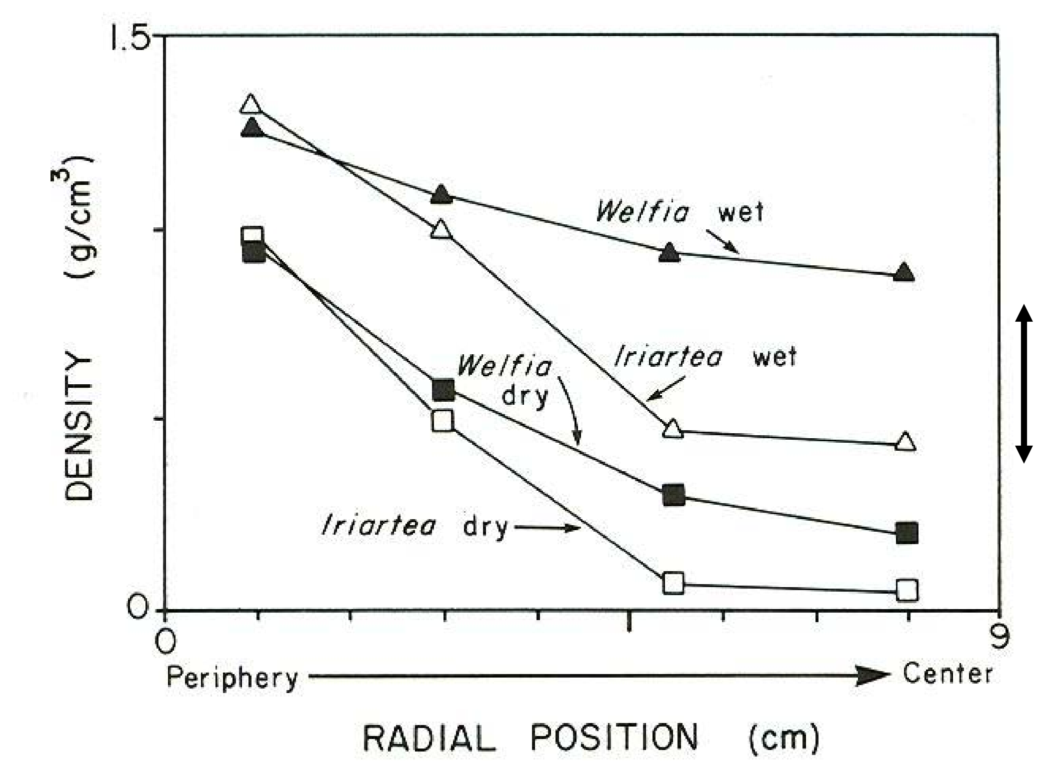
Old

Palm Stem: Density Gradient



Rich, PM (1987) Bot.Gazette <u>148</u>, 42-50.

Palm Stem:
Density at Breast Height

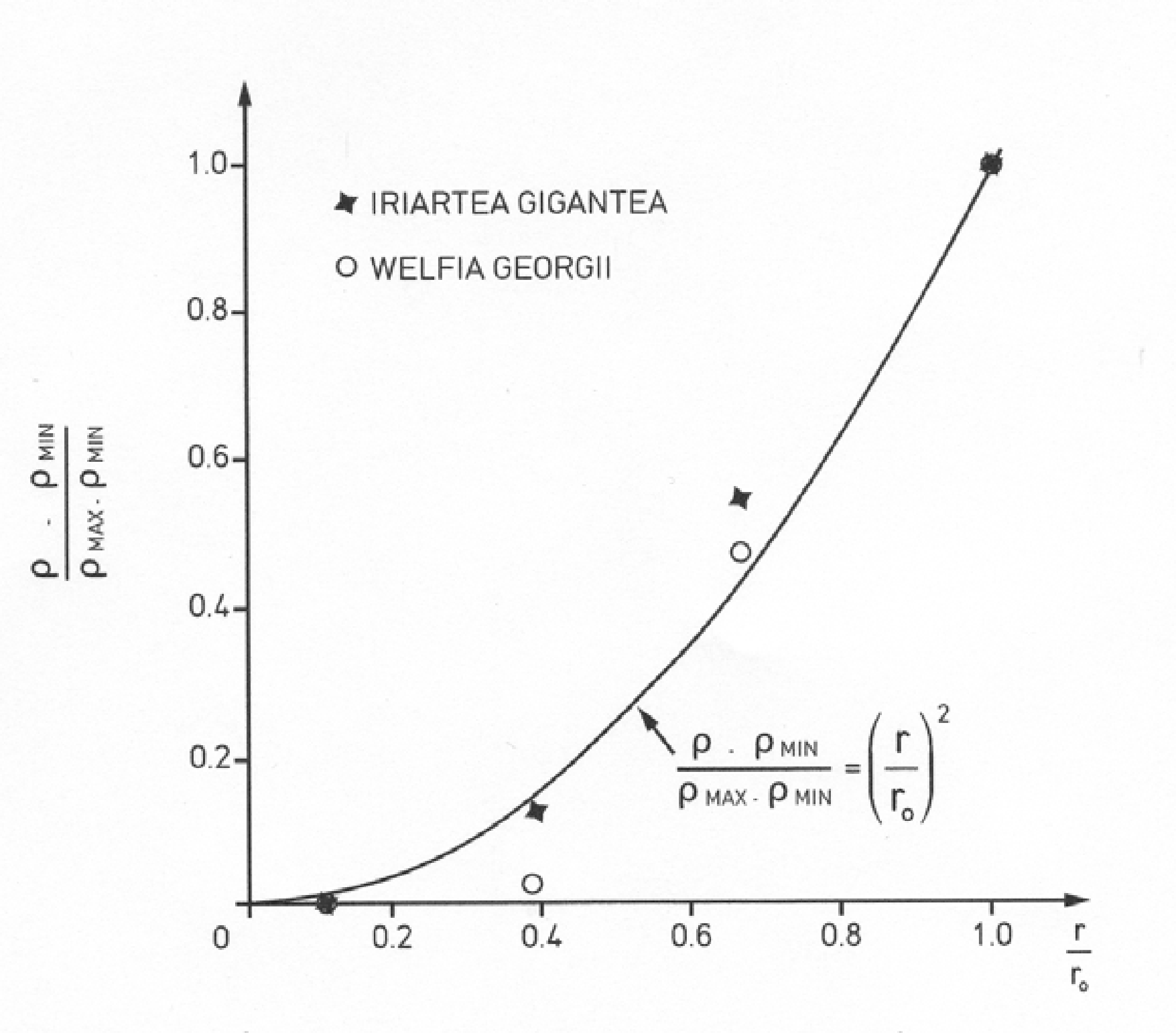


Densities
of common
woods

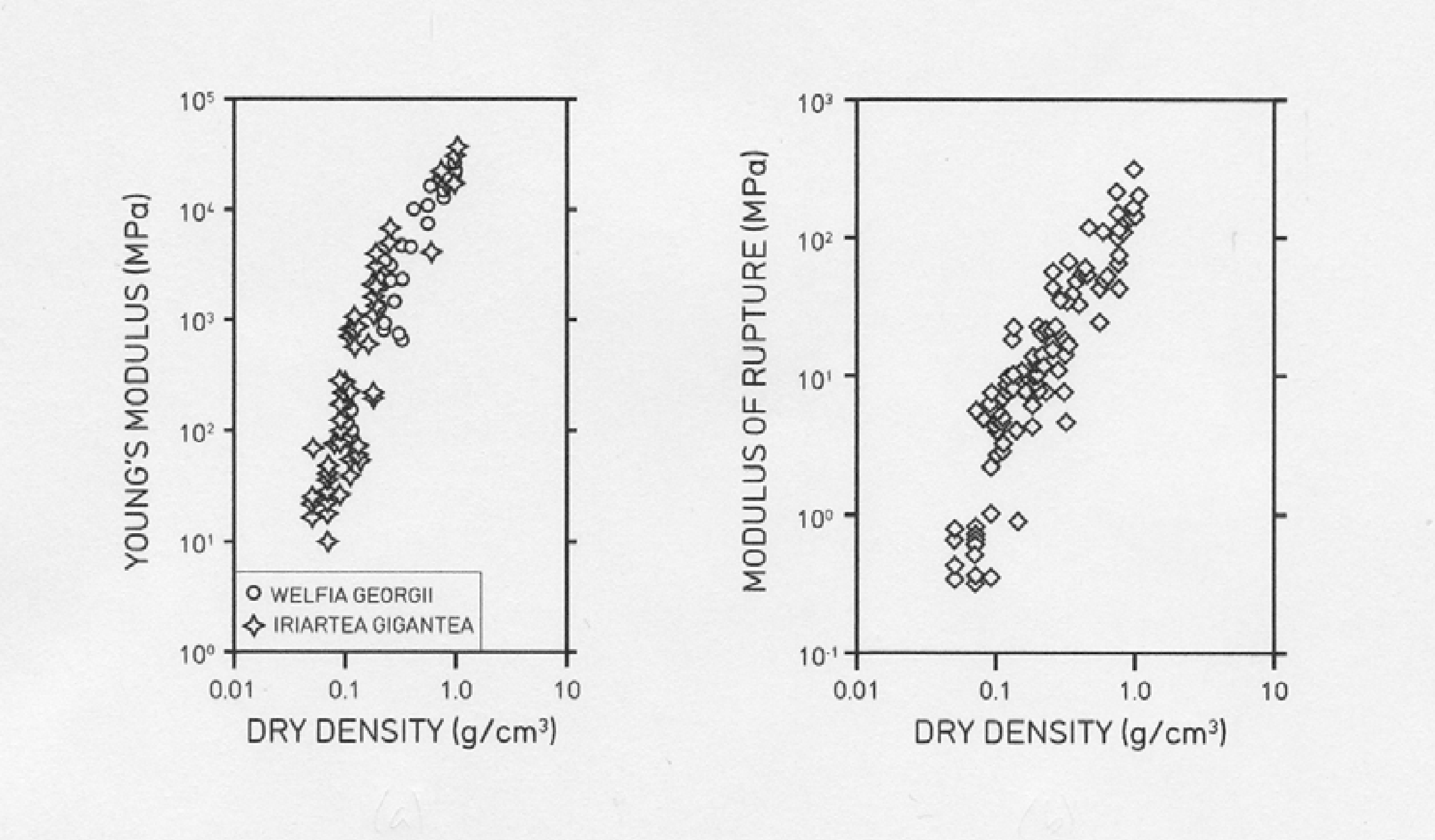
A single mature palm has a similar range of density as nearly all species of wood combined

Rich, PM (1987) Bot.Gazette 148, 42-50.

Palm Stem: Density Gradient



Palm Stem: Mechanical Properties vs. Density



Rich, PM (1987) Bot.Gazette <u>148</u>, 42-50.

Density Gradient:
Iriartea gigantea

$$\rho = \left(\frac{r}{r_o}\right)^n \rho_{\max}$$

$$(EI)_{gradient} = \frac{C\pi r_o^4}{mn+4}$$

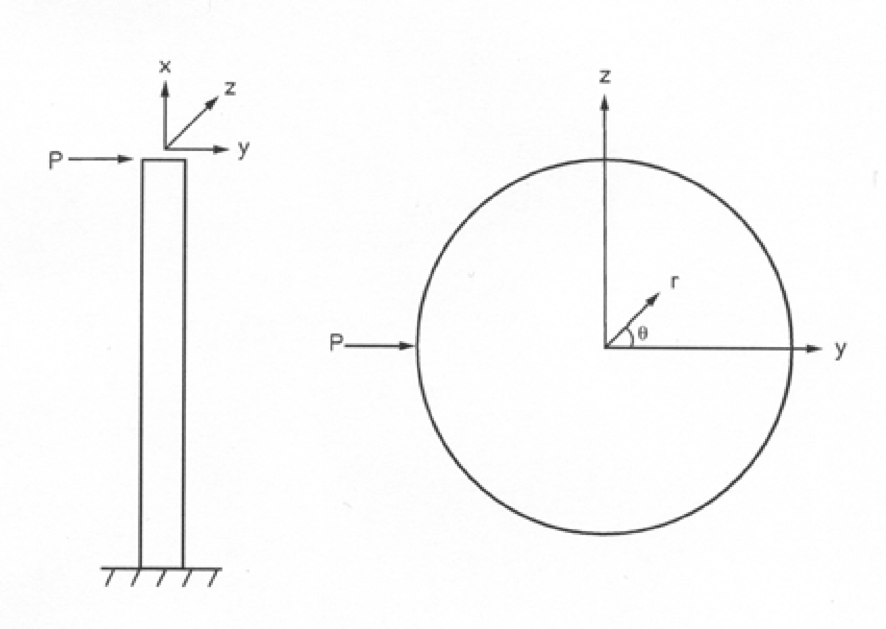
$$E = C\left(\frac{\rho}{\rho_{\max}}\right)^m = C\left(\frac{r}{r_o}\right)^{mn}$$

$$\frac{(EI)_{gradient}}{(EI)_{uniform}} = \frac{4}{mn+4}\left(\frac{n+2}{2}\right)^m$$

Iriartea palm: $n = 2$, $m = 2.5$, $(EI)_{gradient}/(EI)_{uniform} = 2.5$

Similar calculation for *Welfia georgii*, gives
$(EI)_{gradient}/(EI)_{uniform} = 1.6$

Palm Stem:
Bending Stress Distribution



$$\sigma(y) = E\varepsilon = E\kappa y$$

$$\sigma(r,\theta) = C\left(\frac{r}{r_o}\right)^{mn} \kappa r \cos\theta \propto r^{mn+1}$$

Iriartea gigantea: m = 2.5, n =2

$$\sigma \propto r^6$$

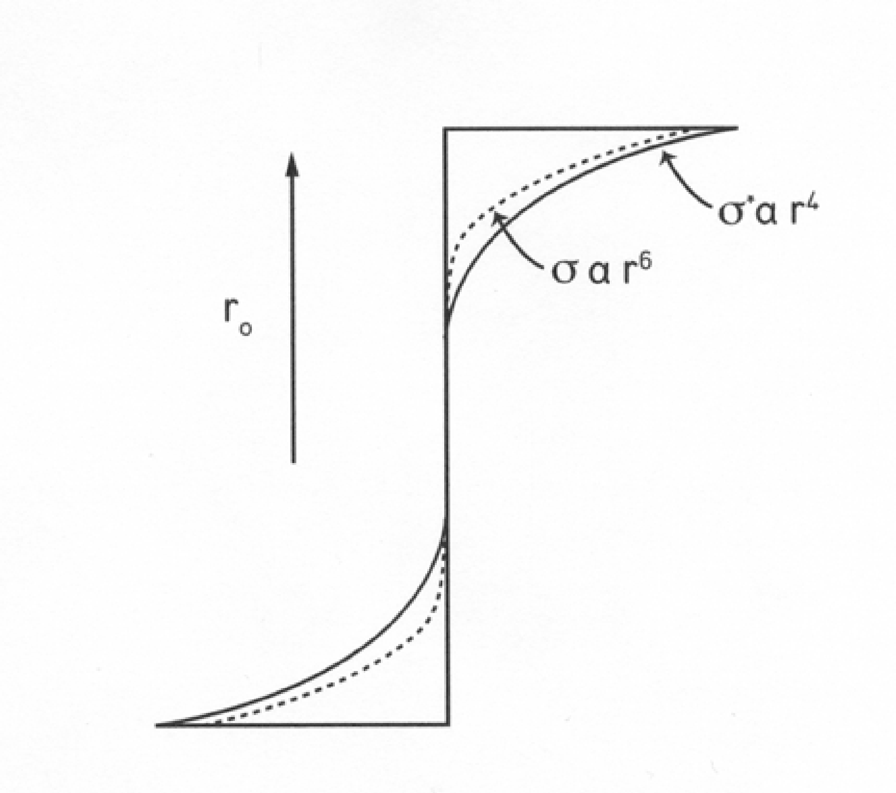
Palm Stem: Bending Strength Distribution

$$\sigma^* \propto \left(\frac{\rho}{\rho_{\mathrm{max}}}\right)^{q} \propto \left(\frac{r}{r_o}\right)^{nq}$$

Iriartea gigantea: n = 2, q = 2

$$\sigma^* \propto r^4$$

Strength matches
bending stress distribution



Plant Stems:
Cylindrical Shells with Compliant Cores

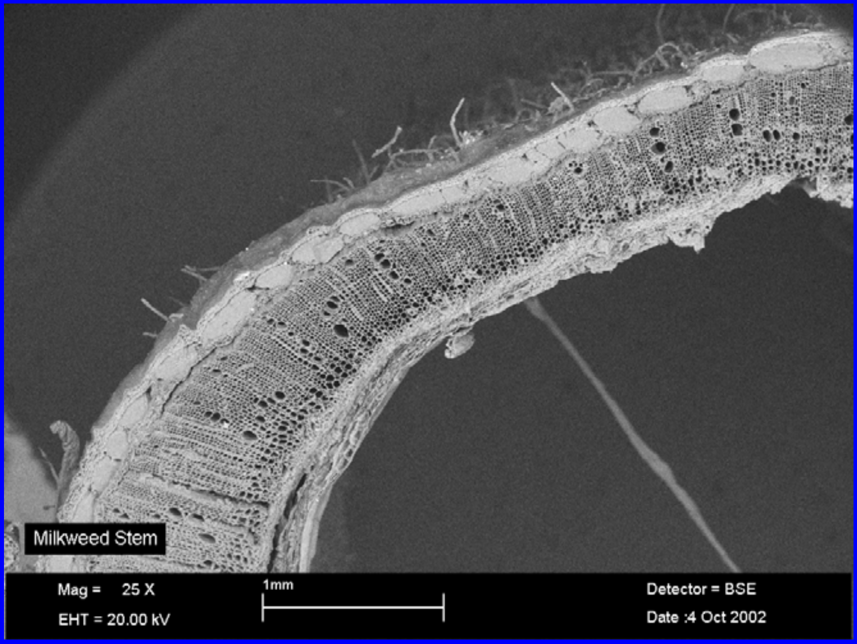
(Also in Animal Quills, Toucan Beak)

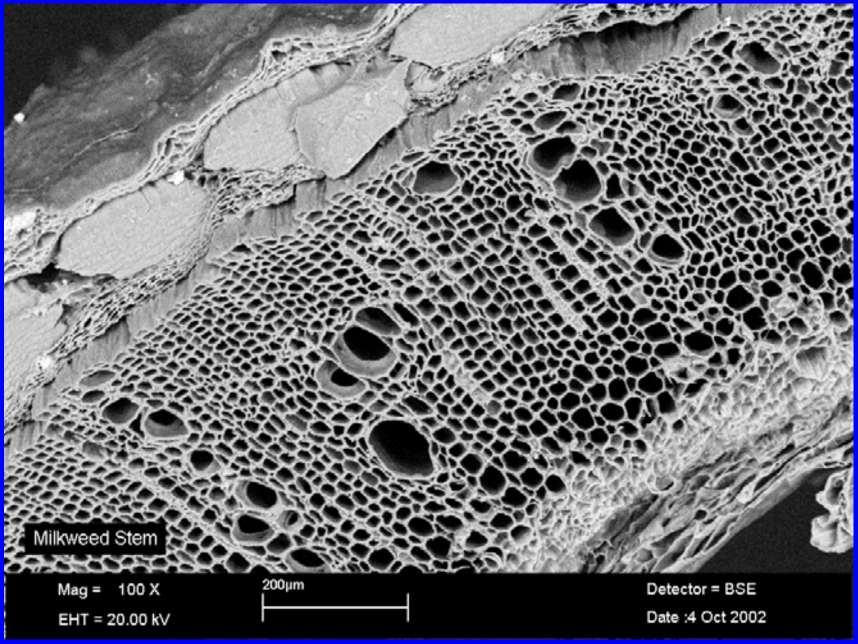
Plant Stems

Milkweed

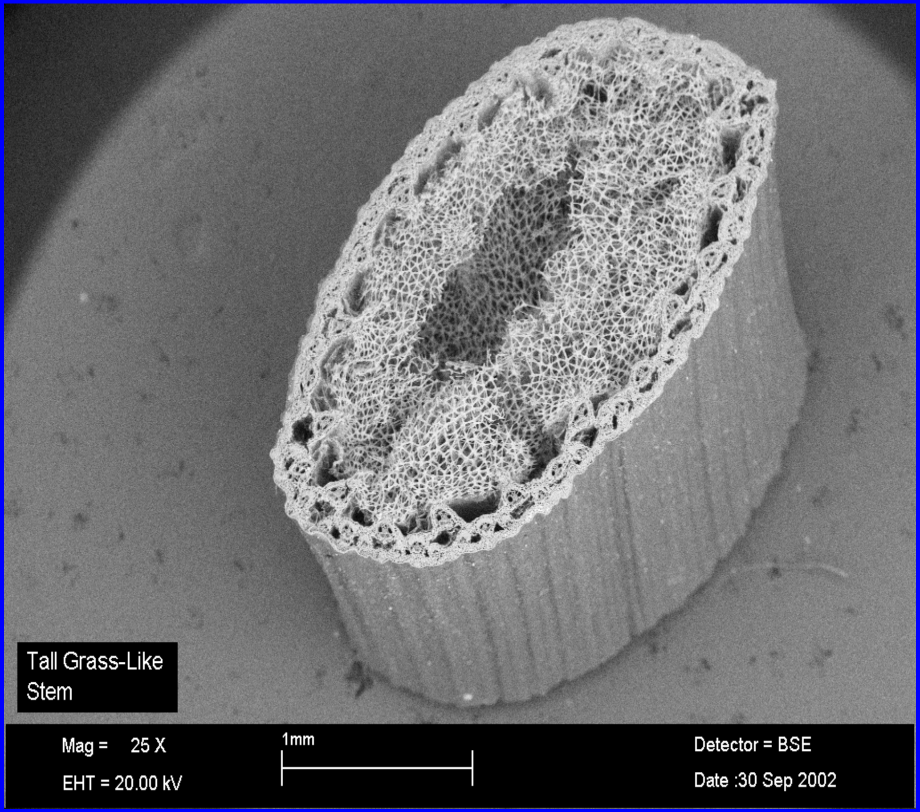
Grassy stem

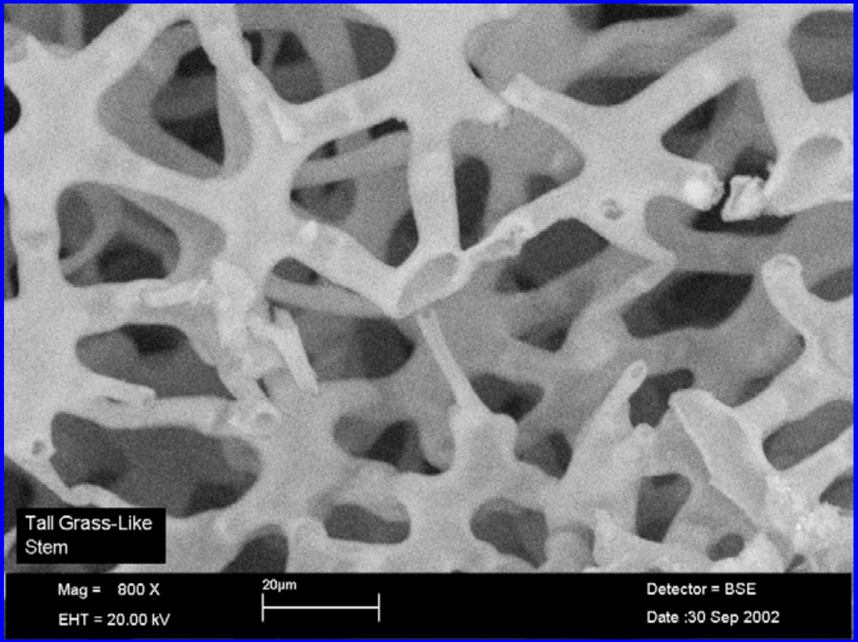
Milkweed Stem





Grassy Stem





Hollow struts

Plant stems

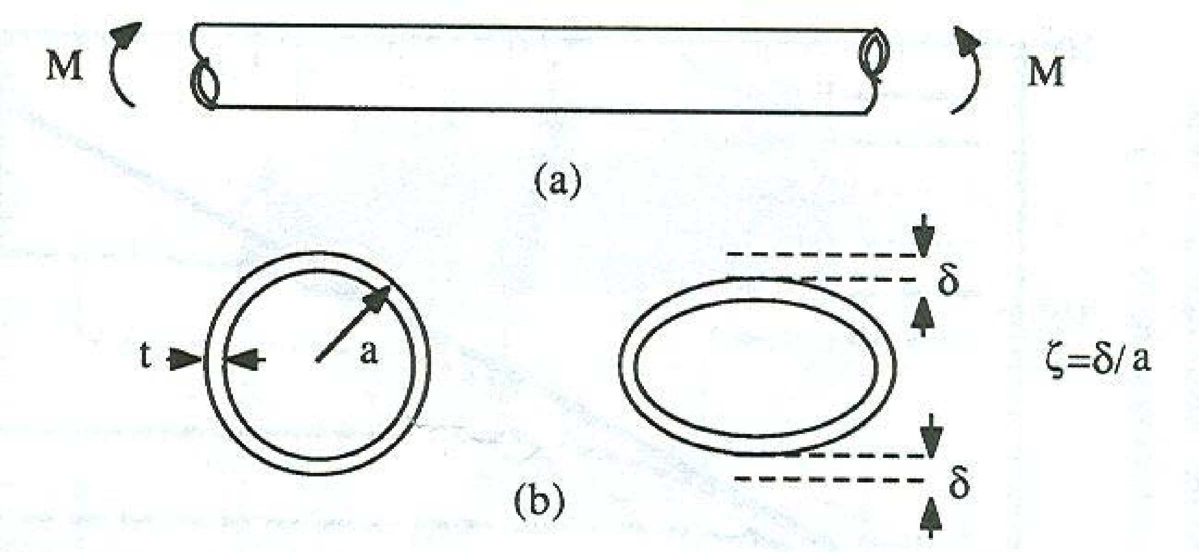
- Circular tube cross-section
- Resists bending (wind loads)
- Maximize shape factor

$$\Phi = \frac{4\pi I}{A^2} = \frac{a}{t}$$

- Maximize a/t, but limited by local buckling and ovalization
- Plant stems have compliant core ("core-rind structure")

Plant Stems: Bending

- Core resists ovalization and increases local buckling resistance



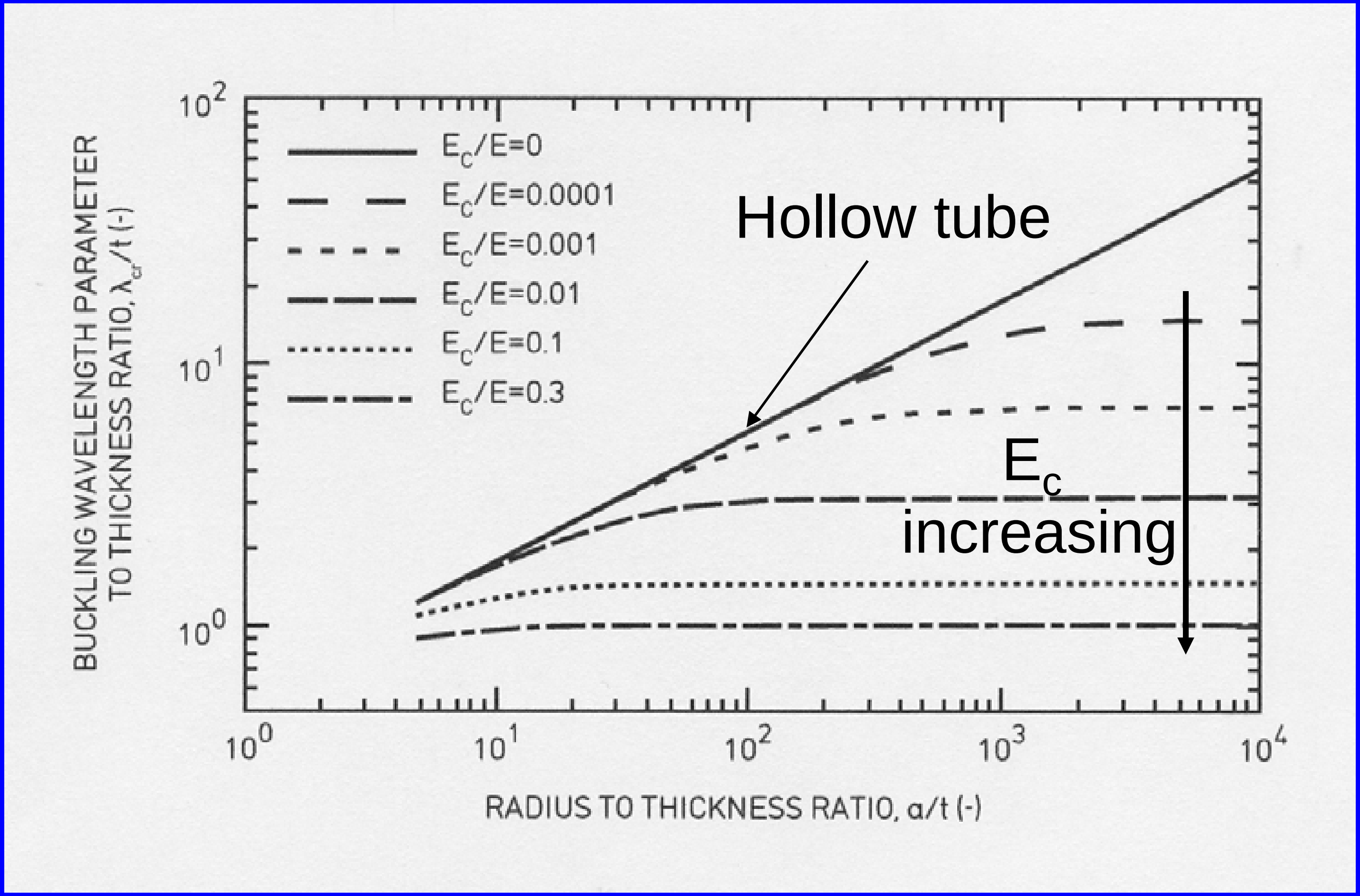
Plant Stems: Bending

- Local buckling occurs when normal stress in compressive side of cylinder equals critical stress for axisymmetric buckling under uniaxial stress
- Hollow cylinder: $M_{lb} = \dfrac{0.939Eat^2}{\sqrt{1-\nu^2}}$
- Cylinder with compliant core:

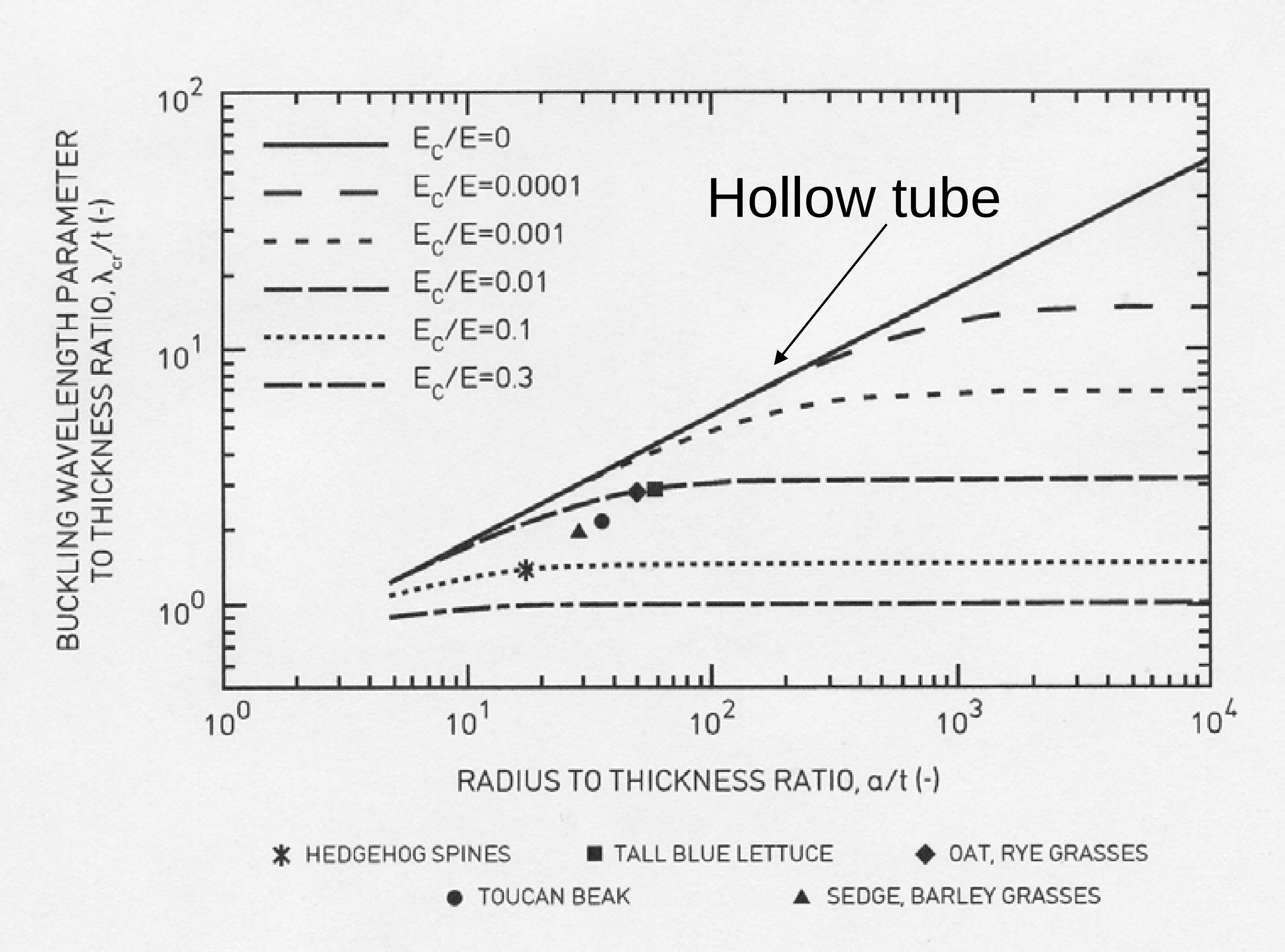
$$M_{lb} = \frac{\pi E a^2 t}{\sqrt{1-\nu^2}} f\left\{\frac{\delta}{r}, \frac{E_{core}}{E_{shell}}, \frac{a}{t}\right\}$$

Plant Stems

- Foam-like core can act like elastic foundation supporting outer shell, increasing local buckling moment, M_{lb}, reducing buckling λ



Plant Stems



10^2
10^1
10^0
BUCKLING WAVELENGTH PARAMETER TO THICKNESS RATIO, λ_{cr}/t (-)
$E_c/E=0$
$E_c/E=0.0001$
$E_c/E=0.001$
$E_c/E=0.01$
$E_c/E=0.1$
$E_c/E=0.3$
Hollow tube
10^0
10^1
10^2
10^3
10^4
RADIUS TO THICKNESS RATIO, a/t (-)
✱ HEDGEHOG SPINES
■ TALL BLUE LETTUCE
◆ OAT, RYE GRASSES
● TOUCAN BEAK
▲ SEDGE, BARLEY GRASSES

Plant Stems

- Within the core stress decays as move radially inward, away from the shell
- Stresses less than 5% of maximum at a radial distance of $5\lambda_{cr}$
- Can remove inner core, leaving core thickness, $c = 5\lambda_{cr}$

Plant Stems

Species	a/t	Elastic foundation	M_{lb}/M_{eq}	c/λ_{cr}
Tall blue lettuce	59	Yes	1.37	3.81
Oat, rye grasses	50	Yes	1.26	3.81
Sedge grass, common barley	25	Yes	0.77	3.27

Core increases buckling resistance for high a/t

Monocotyledon Leaves: Sandwich Structures

(Also in Skulls, Cuttlefish Bone, Horseshoe Crab Shell)

Monocotyledon Leaves: Sandwich Beams

Iris

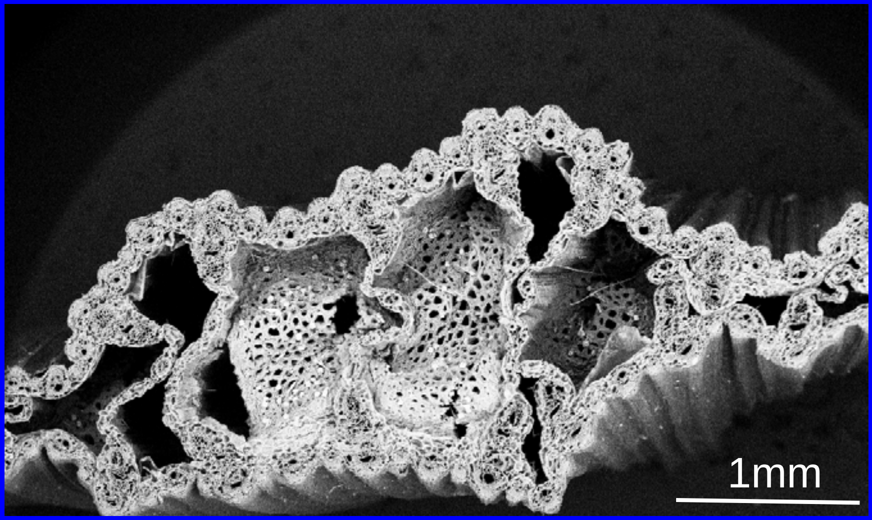
Bulrush

Sandwich Structures: Leaves

Sclerenchyma

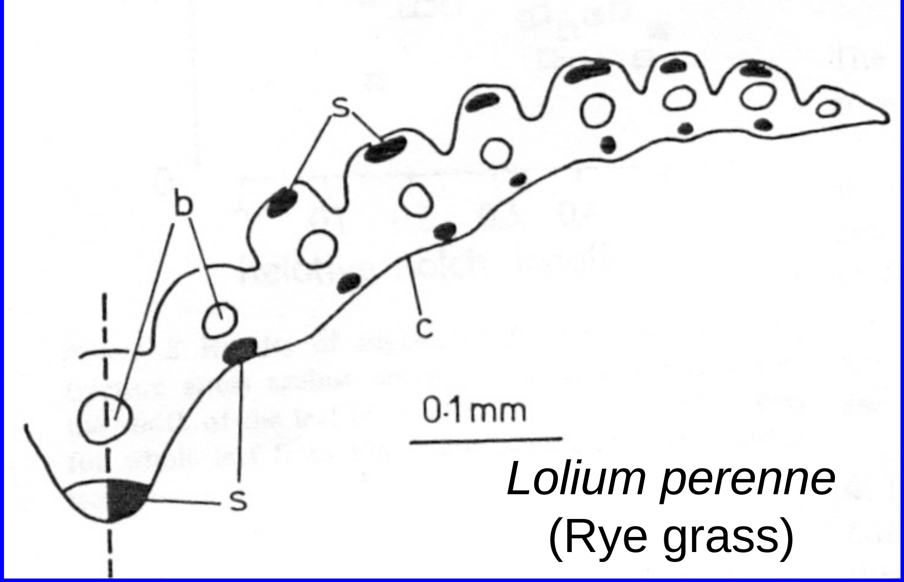
Parenchyma

Iris leaf

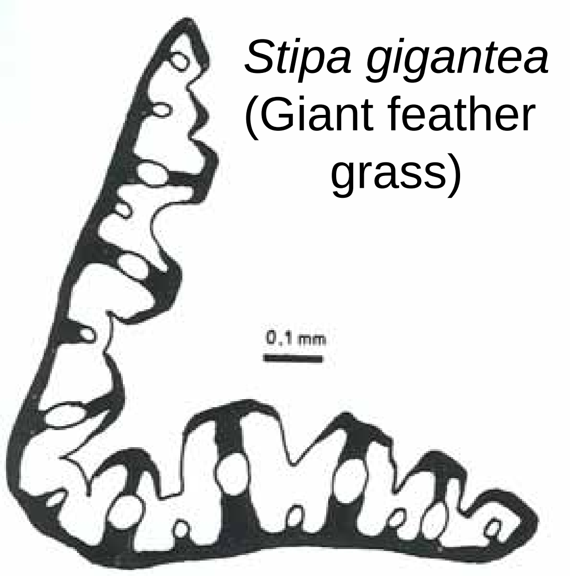


Bulrush leaf

Sandwich Structures: Leaves



Lolium perenne
(Rye grass)



Stipa gigantea
(Giant feather grass)

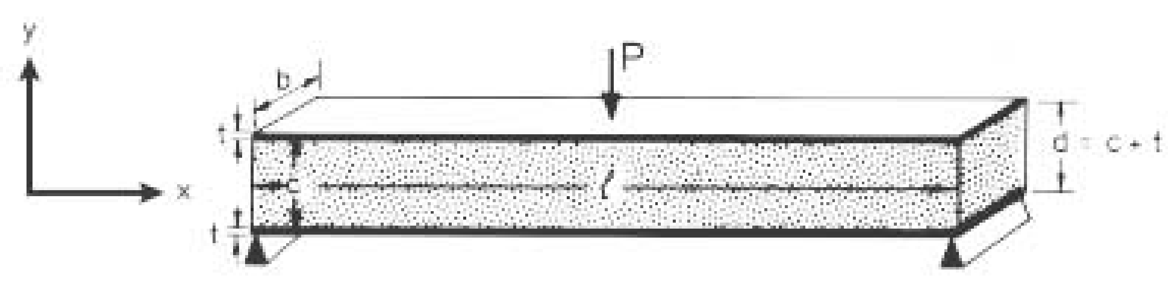
Black = sclerenchyma
White = parenchyma

Vincent, 1982,1991

Monocotyledon Leaves

- Fibers (sclerenchyma) along outer surface of leaves
- Foam-like cells (parenchyma) or ribs in core
- Acts like structural sandwich panel
- Increase in moment of inertia by separating stiff “faces” by a lightweight “core”
- Large surface area for photosynthesis

Sandwich Beam Deflection



$$\delta = \delta_b + \delta_s = \frac{P\mathsf{l}^3}{B_1 (EI)_{eq}} + \frac{P\mathsf{l}}{B_2 (AG)_{eq}}$$

Flexural rigidity: $(EI)_{eq} \approx \dfrac{E_f btc^2}{2}$ Shear rigidity: $(AG)_{eq} \approx bcG_c$

Cantilever: $B_1 = 3$ $B_2 = 1$

Iris Leaves

$t = 30\ \mu m$

$c = 0.5$ to 3.0 mm

$E_f = 8.2$ GPa

$G_c = 2$ MPa

Measured stiffnesses (N/mm):	0.66	0.54	0.41	0.25
Calculated stiffnesses (N/mm):	1.21	0.78	0.51	0.29
Calculated/measured:	1.83	1.44	1.24	1.16

Conclusion

- Wood
  - Uniform honeycomb increases $E^{1/2}/\rho$, $\sigma_f^{2/3}/\rho$
  - Constant ρ, E, σ_f, vary r(z) in tree
- Palm stem
  - Radial density gradient
  - Constant r, vary $\rho(r)$, E(r), $\sigma_f(r)$ in palm stem
  - Increases (EI) relative to uniform distribution of solid
  - Stress distribution across radius matches strength distribution

Conclusion

- Plant stems
  - Cylindrical shell with compliant core
  - Increases buckling resistance over equivalent hollow circular tube for large a/t
- Monocotyledon leaves
  - Sandwich structure, efficient in bending
  - Leaves provide own structural support as well as area for photosynthesis
  - Rectangular cross-section maximizes surface area for photosynthesis

Acknowledgements

- Mike Ashby, Ken Easterling, Hugh Shercliff
- Gebran Karam, Phoebe Cheng, Ulrike Wegst, Ros Olive, Tessa Shercliff
- Justin Breucop, Don Galler, Beth Beighlie

- National Science Foundation
- Matoula S. Salapatas Professorship at MIT

Image References

- Connor S (1994) New England Natives: A celebration of people and trees. Harvard University Press.
- Dinwoodie J (1981)Timber: Its nature and behaviour. Van Nostrand Reinhold.
- Rich PM (1987) Mechanical structure of the stem of arborescent palms. Bot. Gazette 148, 42.
- Rich PM (1987) Developmental anatomy of the stem of Welfia Georgii , Iriartea Gigantea and other arborescent palms: Implications for mechanical support. Am. J. Botany 74, 792.